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DETERMINATION OF THE DETECTION LIMIT AND DECISION THRESHOLD FOR IONIZING-RADIATION MEASUREMENTS: FUNDAMENTALS AND PARTICULAR APPLICATIONS

Proposal for a standard

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Prologue

The recognition and detection of ionizing radiation are indispensable basic prerequisites of radiation protection. For this purpose, the standard series DIN 25482 and the corresponding standard series ISO 11929 provide decision thresholds, detection limits, and confidence limits for a diversity of application fields. The decision threshold allows a decision to be made for a measurement on whether or not, for instance, radiation of a possibly radioactive sample is present. The detection limit allows a decision on whether or not the measurement procedure intended for application to the measurement meets the requirements to be fulfilled and is therefore appropriate for the measurement purpose. Confidence limits enclose with a specified probability the true value of the measurand to be measured.

Because of recent developments in metrology concerning measurement uncertainty (DIN 1319 and ISO Guide to the expression of uncertainty in measurement), the older Parts 1 to 7 (except Part 4) of DIN 25482 and the corresponding Parts 1 to 4 of ISO 11929 urgently need a revision based on the common, already laid statistical foundation of Part 10 of DIN 25482. The modern Parts 11 to 13 of DIN 25482 and Parts 5 to 8 of ISO 11929 are already established on this basis. But since the responsible working group DIN NMP 722 was first suspended and finally disbanded by DIN, the authors, feeling responsible for radiation protection and being members of the working group "Detection limits" (AK SIGMA) of the German Radiation Protection Association (Fachverband für Strahlenschutz), elaborated the present standard proposal. This proposal represents a new version of the mentioned older parts and unifies them on the basis of the general Part 10 of DIN 25482 and Part 7 of ISO 11929 for a diversity of particular applications to measurements of ionizing radiation.

The original, first published German edition of the elaborated standard proposal (Nachweisgrenze und Erkennnungsgrenze bei Kernstrahlungsmessungen: Spezielle Anwendungen – Vorschlag für eine Norm. FS-04-127-AKSIGMA, Fachverband für Strahlenschutz, TÜV-Verlag, Cologne, 2004, ISBN 3-8249-0904-9) was designed in a form that could immediately be published with only minor changes as a DIN draft standard as soon as the responsible working group will be revived. It should then be proposed with the new number DIN 25482-1 to replace the presently still valid standards DIN 25482-1:1989-04, DIN 25482-2:1992-09, DIN 25482-3:1993-02, DIN 25482-5:1993-06, DIN 25482-6:1993-02, DIN 25482-7:1997-12, and possibly also DIN 25482-13:2003-02. Likewise, the present English translation of the standard proposal could more or less directly serve for revising, unifying and replacing the standards ISO 11929 Parts 1 to 4.

Contents

Foreword

This standard proposal has been elaborated by the working group "Detection limits" (AK SIGMA) of the German Radiation Protection Association.

Annexes A to C are normative, Annexes D to G are informative. DIN 25482 "Detection limit and decision threshold for ionizing radiation measurements" should, on the basis of this standard proposal, in future consist of:

- Part 1: Particular applications
- Part 10: General applications
- Part 11: Measurements using albedo dosimeters
- Part 12: Unfolding of spectra
- Part 13: Counting measurements on moving objects

Likewise, ISO 11929 "Determination of the detection limit and decision threshold for ionizing radiation measurements" should, also on the basis of this standard proposal, in future consist of:

- Part 1: Fundamentals and particular applications
- Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material
- Part 6: Fundamentals and applications to measurements by use of transient mode
- Part 7: Fundamentals and general applications
- Part 8: Fundamentals and application to unfolding of spectrometric measurements without the influence of sample treatment

Amendments

DIN 25482-1, DIN 25482-2, DIN 25482-3, DIN 25482-5, DIN 25482-6 and DIN 25482-7, on the one hand, and ISO 11929-1, ISO 11929-2, ISO 11929-3 and ISO 11929-4, on the other hand, have been unified and rewritten on the basis of Bayesian statistics, DIN 25482-10 and ISO 11929-7.

Previous editions

DIN 25482-1: 1989-04, DIN 25482-2: 1992-09, DIN 25482-3: 1993-02, DIN 25482-5: 1993-06, DIN 25482-6: 1993-02, DIN 25482-7: 1997-12, ISO 11929-1: 2000, ISO 11929-2: 2000, ISO 11929-3: 2000, ISO 11929-4: 2001. The standard DIN 25482-4: 1995-12 missing here is incorporated into DIN 25482-12: 2003-02.

Introduction

The limits to be provided according to the present standard proposal by means of statistical tests and specified probabilities allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

- The *decision threshold* allows a decision on whether or not the physical effect quantified by the measurand is present.
- The *detection limit* indicates which smallest true value of the measurand can still be detected with a measurement procedure to be applied. This allows a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose.
- The *confidence limits* enclose, in the case of the physical effect being recognized as present, a confidence interval containing the true value of the measurand with a specified probability.

In the following, the mentioned limits are jointly called *characteristic limits*.

This standard proposal is based on DIN 25482-10 and ISO 11929-7 and thus on procedures of Bayesian statistics (see [3], [4], [5], [6], [7]), so that uncertain quantities and influences can also be taken into account, which do not behave randomly in measurements repeated several times or in counting measurements. Since measurement uncertainty plays an important part in this standard proposal, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to DIN 1319-3, DIN 1319-4, DIN V ENV 13005, [1] or [3]. This enables the strict separation of the evaluation of the measurements, on the one hand (Section 5), and the provision and calculation of the characteristic limits, on the other hand (Section 6).

Equations are provided for the calculation of the characteristic limits of an ionizing-radiation measurand via the *standard measurement uncertainty* of the measurand (called *standard uncertainty* in the following). The standard uncertainties of the measurement as well as those of sample treatment, calibration of the measuring system and other influences are taken into account. But the latter standard uncertainties are assumed to be known from previous investigations.

Determination of the detection limit and decision threshold for ionizing-radiation measurements — Fundamentals and particular applications

1 Scope

The present standard proposal applies in the field of ionizing-radiation metrology to the provision of the *decision threshold*, the *detection limit*, and the *confidence limits* for a non-negative ionizing-radiation measurand when counting measurements with preselection of time or counts are carried out, and the measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment, and by other factors.

The present standard proposal also applies in the same way to

- counting measurements on moving objects (DIN 25482-13 and ISO 11929-6, see B.2),
- measurements with linear-scale analogue count rate measuring instruments (called ratemeters in the following, see B.3),
- repeated counting measurements with random influences (see B.4),
- counting measurements on filters during accumulation of radioactive material (ISO 11929-5, see B.5),
- counting spectrometric multi-channel measurements if particular lines in the spectrum are to be considered and no adjustment calculations, for instance, an unfolding (DIN 25482-12 and ISO 11929-8), have to be carried out (see Annex C).

The present standard proposal also applies analogously to other measurements of any kind if the same model of the evaluation is involved.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this standard proposal. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this standard proposal are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

DIN 1313 Größen

DIN 1319-1 Grundlagen der Messtechnik – Teil 1: Grundbegriffe

DIN 1319-3 Grundlagen der Messtechnik – Teil 3: Auswertung von Messungen einer einzelnen Messgröße, Messunsicherheit

DIN 1319-4 Grundlagen der Messtechnik – Teil 4: Auswertung von Messungen, Messunsicherheit

DIN 13303-1 Stochastik – Wahrscheinlichkeitstheorie, Gemeinsame Grundbegriffe der mathematischen und der beschreibenden Statistik, Begriffe und Zeichen

DIN 13303-2 Stochastik – Mathematische Statistik, Begriffe und Zeichen

DIN 25482-10 Nachweisgrenze und Erkennungsgrenze bei Kernstrahlungsmessungen – Teil 10: Allgemeine Anwendungen

DIN 25482-12 Nachweisgrenze und Erkennungsgrenze bei Kernstrahlungsmessungen – Teil 12: Entfaltung von Spektren

DIN 25482-13 Nachweisgrenze und Erkennungsgrenze bei Kernstrahlungsmessungen – Teil 13: Zählende Messungen an bewegten Objekten

DIN 53804-1 Statistische Auswertungen – Messbare (kontinuierliche) Merkmale

DIN 55350-12 Begriffe der Qualitätssicherung und Statistik – Merkmalsbezogene Begriffe

DIN 55350-21 Begriffe der Qualitätssicherung und Statistik – Begriffe der Statistik, Zufallsgrößen und Wahrscheinlichkeitsverteilungen

DIN 55350-22 Begriffe der Qualitätssicherung und Statistik – Begriffe der Statistik, Spezielle Wahrscheinlichkeitsverteilungen

DIN 55350-23 Begriffe der Qualitätssicherung und Statistik – Begriffe der Statistik, Beschreibende Statistik

DIN 55350-24 Begriffe der Qualitätssicherung und Statistik – Begriffe der Statistik, Schließende Statistik

DIN V ENV 13005 Leitfaden zur Angabe der Unsicherheit beim Messen – Deutsche Fassung ENV 13005

ISO 31-0 Quantities and units – Part 0: General principles

ISO 31-9 Quantities and units – Part 9: Atomic and nuclear physics

ISO 3534-1 Statistics – Vocabulary and symbols – Part 1: Probability and general statistical terms

ISO 11929-5 Determination of the detection limit and decision threshold for ionizing radiation measurements – Part 5: Fundamentals and applications to counting measurements on filters during accumulation of radioactive material

ISO 11929-6 Determination of the detection limit and decision threshold for ionizing radiation measurements – Part 6: Fundamentals and applications to measurements by use of transient mode

ISO 11929-7 Determination of the detection limit and decision threshold for ionizing radiation measurements – Part 7: Fundamentals and general applications

ISO 11929-8 Determination of the detection limit and decision threshold for ionizing radiation measurements – Part 8: Fundamentals and application to unfolding of spectrometric measurements without the influence of sample treatment

[1] Guide to the Expression of Uncertainty in Measurement. ISO International Organization for Standardization (Geneva) 1993, corrected ed. 1995, also as ENV 13005:1999

[2] International Vocabulary of Basic and General Terms in Metrology. ISO International Organization for Standardization (Geneva) 1993; Internationales Wörterbuch der Metrologie – International Vocabulary of Basic and General Terms in Metrology. DIN Deutsches Institut für Normung (Ed.), Beuth Verlag (Berlin, Cologne) 1994

[3] K. Weise, W. Wöger: Messunsicherheit und Messdatenauswertung. Wiley-VCH (Weinheim) 1999

[4] P.M. Lee: Bayesian Statistics: An Introduction. Oxford University Press (New York) 1989

[5] D. Wickmann: Bayes-Statistik. Mathematische Texte, Vol. 4, Eds.: N. Knocke, H. Scheid, BI Wissenschaftsverlag, Bibliographisches Institut and F.A. Brockhaus (Mannheim, Vienna, Zürich) 1990

[6] K. Weise, W. Wöger: Eine Bayessche Theorie der Messunsicherheit. PTB Report N–11, Physikalisch-Technische Bundesanstalt (Braunschweig) 1992; A Bayesian theory of measurement uncertainty. Meas. Sci. Technol. 4; 1–11; 1993

[7] K. Weise: Bayesian-statistical decision threshold, detection limit and confidence interval in nuclear radiation measurement. Kerntechnik 63; 214–224; 1998

[8] F. Kohlrausch: Praktische Physik. 24th ed., Vol. 3, p. 613. B.G. Teubner (Stuttgart) 1996

[9] M. Abramowitz, I. Stegun: Handbook of Mathematical Functions. 5th ed., Chap. 26, Dover Publications (New York) 1968

[10] K. Weise: The Bayesian count rate probability distribution in measurement of ionizing radiation by use of a ratemeter. PTB Report Ra-44, Physikalisch-Technische Bundesanstalt (Braunschweig) 2004

3 Terms

For the application of this standard proposal, the definitions given by DIN 1319-1, DIN 1319-3, DIN 1319-4, DIN 13303-1, DIN 13303-2, DIN 25482-10, DIN 53804-1, by the standards of the DIN 55350 series listed in Section 2, by DIN V ENV 13005, ISO 31-0, ISO 31-9, ISO 3534-1, ISO 11929-7, and [2] shall apply. In addition, the terms informatively given in Annex F are used.

4 Quantities and symbols

The symbols for auxiliary quantities and the symbols only used in the annexes are not listed.

m	number of the input quantities
X_i	input quantity $(i = 1, \ldots, m)$
x_i	estimate of the input quantity X_i
$u(x_i)$	standard uncertainty of the input quantity X_i associated with the estimate x_i
$h_1(x_1)$	standard uncertainty $u(x_1)$ as a function of the estimate x_1
Δx_i	width of the region of the possible values of the input quantity X_i
$u_{\mathrm{rel}}(w)$	relative standard uncertainty of a quantity W associated with the estimate w
G	model function
Y	random variable as an estimator of the measurand; also used as the symbol for the non-negative measurand itself, which quantifies the physical effect of interest
η	true value of the measurand. If the physical effect of interest is not present, then $\eta = 0$, otherwise, $\eta > 0$.
y	determined value of the estimator Y; primary measurement result of the measurand
y_j	values y from different measurements $(j = 0,1,2,\ldots)$
$u(y)$	standard uncertainty of the measurand associated with the primary measurement result y
$\widetilde{u}(\eta)$	standard uncertainty of the estimator Y as a function of the true value η of the measurand
z	best estimate of the measurand
$u(z)$	standard uncertainty of the measurand associated with the best estimate z
y^*	decision threshold of the measurand
η^*	detection limit of the measurand
η_i	approximations of the detection limit η^*
η_{r}	guideline value of the measurand
η_{l}, η_{u}	lower and upper confidence limit, respectively, of the measurand
ϱ_i	count rate as an input quantity X_i
ϱ_{n}	count rate of the net effect (net count rate)
ϱ_{g}, ϱ_0	count rate of the gross effect (gross count rate) and of the background effect (background count rate), respectively
n_i	number of the counted pulses obtained from the measurement of the count rate ϱ_i
n_{g}, n_0	number of the counted pulses of the gross effect and of the background effect, respectively
t_i	measurement duration of the measurement of the count rate ϱ_i
t_{g}, t_0	measurement duration of the measurement of the gross effect and of the background effect, respectively
r_i	estimate of the count rate ϱ_i
r_{g}, r_0	estimate of the gross count rate and of the background count rate, respectively
τ_{g}, τ_0	relaxation time constant of a ratemeter used for the measurement of the gross effect and of the background effect, respectively
α, β	probability of the error of the first and second kind, respectively
$1-\gamma$	probability for the confidence interval of the measurand
k_p, k_q	quantiles of the standardized normal distribution for the probabilities p and q, respectively (for instance, $p = 1-\alpha$, $1-\beta$ or $1-\gamma/2$)
$\Phi(t)$	distribution function of the standardized normal distribution. $\Phi(k_p) = p$ applies.

5 Fundamentals

5.1 General aspects concerning the measurand

A non-negative measurand must be assigned to the physical effect to be investigated according to a given measurement task. The measurand has to quantify the effect and to assume the true value $\eta = 0$ if the effect is not present in a particular case.

Then, a random variable Y, an estimator, must be assigned to the measurand. The symbol Y is also used in the following for the measurand itself. A value y of the estimator Y, determined from measurements, is an estimate of the measurand. It has to be calculated as the primary measurement result together with the primary standard uncertainty $u(y)$ of the measurand associated with y. Both values form the primary complete measurement result for the measurand and are obtained according to DIN 1319-3, DIN 1319-4, DIN V ENV 13005, [1] or [3] by evaluation of the measurement data and other information by means of a model (of the evaluation), which mathematically connects all the quantities involved (see 5.2). In general, the fact that the measurand is non-negative is not explicitly taken into account in the evaluation. Therefore, y may be negative, especially when the measurand nearly assumes the true value $\eta = 0$. The primary measurement result y differs from the best estimate z of the measurand calculated in 6.5. With z, the knowledge that the measurand is non-negative is taken into account. The standard uncertainty $u(z)$ associated with z is smaller than $u(y)$.

NOTE The best estimate among all possible estimates of the measurand on the basis of given information is associated with the minimum standard uncertainty.

5.2 Model

5.2.1 General model

In many cases, the measurand Y is a function of several input quantities X_i in the form of

$$Y = G(X_1, \dots, X_m) \ . \tag{1}$$

Equation (1) is the model of the evaluation. Substituting given estimates x_i of the input quantities X_i in the model function G of equation (1) yields the primary measurement result y of the measurand as

$$y = G(x_1, \dots, x_m) \ . \tag{2}$$

The standard uncertainty $u(y)$ of the measurand associated with the primary measurement result y follows, if the input quantities X_i are independently measured and standard uncertainties $u(x_i)$ associated with the estimates x_i are given, from the relation

$$u^2(y) = \sum_{i=1}^{m} \left(\frac{\partial G}{\partial X_i} \right)^2 u^2(x_i) \ . \tag{3}$$

The estimates x_i have to be substituted for the input quantities X_i in the partial derivatives of G in equation (3). For the determination of the estimates x_i and the associated standard uncertainties $u(x_i)$ and also for the numerical or experimental determination of the partial derivatives, see DIN 1319-3, DIN 1319-4, DIN V ENV 13005, DIN 25482-10, ISO 11929-7, [1] or [3]. For a count rate $X_i = \varrho_i$ with the given counting result n_i recorded during the measurement of duration t_i, the specifications $x_i = r_i = n_i/t_i$ and $u^2(x_i) = n_i/t_i^2 = r_i/t_i$ apply (see also G.1).

In the following, the input quantity X_1, for instance, the gross count rate, is taken as that quantity whose value x_1 is not given when a true value η of the measurand Y is specified within the framework of the calculation of the decision threshold and the detection limit. Analogously, the input quantity X_2 is assigned in a suitable way to the background effect. The data of the other input quantities are taken as given from independent previous investigations.

5.2.2 Model in ionizing-radiation measurements

In this standard proposal, the measurand Y with its true value η relates to a sample of radioactive material and is to be determined from countings of the gross effect and of the background effect with preselection of time or counts. In particular, Y can be the net count rate ϱ_{n} or the net activity A of the sample. The symbols belonging to the countings of the gross effect and of the background effect are marked in the following by the subscripts g and 0, respectively.

In this standard proposal, the model is specified as follows:

$$Y = G(X_1, \dots, X_m) = (X_1 - X_2 X_3) \cdot \frac{X_4 X_6 \cdots}{X_5 X_7 \cdots} = (X_1 - X_2 X_3) \cdot W \tag{4}$$

with the abbreviation

$$W = \frac{X_4 X_6 \cdots}{X_5 X_7 \cdots} \ . \tag{5}$$

$X_1 = \varrho_g$ is the gross count rate and $X_2 = \varrho_0$ is the background count rate. The other input quantities X_i are calibration, correction or influence quantities, or conversion factors, for instance, the emission or response probability or, in particular, X_3 is a shielding factor. If some of these input quantities are not involved, $x_i = 1$ and $u(x_i) = 0$ must be set for them. For the count rates, $x_1 = r_g = n_g/t_g$ and $u^2(x_1) = n_g/t_g^2 = r_g/t_g$ as well as $x_2 = r_0 = n_0/t_0$ and $u^2(x_2) = n_0/t_0^2 = r_0/t_0$ apply.

By substituting the estimates x_i in equation (4), the primary estimate y of the measurand Y results:

$$y = G(x_1, \ldots, x_m) = (x_1 - x_2 x_3) \cdot w = (r_g - r_0 x_3) \cdot w = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0} x_3\right) \cdot w \tag{6}$$

with the abbreviation

$$w = \frac{x_4 x_6 \cdots}{x_5 x_7 \cdots} \ . \tag{7}$$

With the partial derivatives

$$\frac{\partial G}{\partial X_1} = W \ ; \quad \frac{\partial G}{\partial X_2} = -X_3 W \ ; \quad \frac{\partial G}{\partial X_3} = -X_2 W \ ; \quad \frac{\partial G}{\partial X_i} = \pm \frac{Y}{X_i} \quad (i \geq 4) \ , \tag{8}$$

and by substituting the estimates x_i, w and y, equation (3) yields the standard uncertainty $u(y)$ of the measurand associated with y:

$$\begin{aligned} u(y) &= \sqrt{w^2 \cdot \left(u^2(x_1) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3)\right) + y^2 u_{\text{rel}}^2(w)} \\ &= \sqrt{w^2 \cdot \left(r_g/t_g + x_3^2 r_0/t_0 + r_0^2 u^2(x_3)\right) + y^2 u_{\text{rel}}^2(w)} \end{aligned} \tag{9}$$

where

$$u_{\text{rel}}^2(w) = \sum_{i=4}^{m} \frac{u^2(x_i)}{x_i^2} \tag{10}$$

is the sum of the squared relative standard uncertainties of the quantities X_4 to X_m. For $m < 4$, the values $w = 1$ and $u_{\text{rel}}^2(w) = 0$ apply.

The estimate x_i and the standard uncertainty $u(x_i)$ of X_i $(i = 3, \ldots, m)$ are taken as determined in previous investigations or as values of experience according to other information. In the previous investigations, x_i can be determined as an arithmetic mean value and $u^2(x_i)$ as an empirical variance (see B.4.1). If necessary, $u^2(x_i)$ can also be calculated as the variance of a rectangular distribution over the region of the possible values of X_i with the width Δx_i. This yields $u^2(x_i) = (\Delta x_i)^2/12$.

For the application of the procedure to particular measurements, including spectrometric measurements, see the normative Annexes B and C.

5.3 Calculation of the standard uncertainty as a function of the measurand

5.3.1 General aspects

For the provision and numerical calculation of the decision threshold in 6.2 and of the detection limit in 6.3, the standard uncertainty of the measurand is needed as a function $\widetilde{u}(\eta)$ of the true value $\eta \geq 0$ of the measurand. This function has to be determined in a way similar to $u(y)$ within the framework of the evaluation of the measurements by application of DIN 1319-3, DIN 1319-4, DIN V ENV 13005, [1] or [3]. In most cases, $\widetilde{u}(\eta)$ has to be formed as a positive square root of a variance function $\widetilde{u}^2(\eta)$ calculated first. This function must be defined, unique and continuous for all $\eta \geq 0$ and must not assume negative values.

In some cases, $\widetilde{u}(\eta)$ can be explicitly specified, provided that $u(x_1)$ is given as a function $h_1(x_1)$ of x_1. In such cases, y has to be replaced by η and equation (2) must be solved for x_1. With a specified η, the value x_1 can also be calculated numerically from equation (2), for instance, by means of an iteration procedure, which results in x_1 as a function of η and $x_2, \ldots, x_m$. This function has to replace x_1 in equation (3) and in $u(x_1) = h_1(x_1)$, which finally yields $\widetilde{u}(\eta)$ instead of $u(y)$. In the case of the model according to equation (6) and 5.3.2, one has to proceed in this way. Otherwise, 5.3.3 must be applied, where $\widetilde{u}(\eta)$ follows as an approximation by interpolation from the data y_j and $u(y_j)$ of several measurements.

5.3.2 Explicit calculation

When, in the case of the model according to equation (6), the standard uncertainty $u(x_1)$ of the gross count rate $X_1 = \varrho_\text{g}$ is given as a function $h_1(x_1)$ of the estimate $x_1 = r_\text{g}$, then either $h_1(x_1) = \sqrt{x_1/t_\text{g}}$ or $h_1(x_1) = x_1/\sqrt{n_\text{g}}$ applies if the measurement duration t_g (time preselection) or, respectively, the number n_g of recorded pulses (preselection of counts) is specified.

The value y has to be replaced by η. This allows the elimination of x_1 in the general case and, in particular, of n_g with time preselection and of t_g with preselection of counts in equation (9) by means of equation (6). These values are not available when η is specified. This yields in the general case according to equation (6)

$$x_1 = \eta/w + x_2 x_3 \ . \tag{11}$$

By substituting x_1 according to equation (11) in the given function $h_1(x_1)$, i.e. with $u^2(x_1) = h_1^2(\eta/w + x_2 x_3)$, the following results from equation (9):

$$\widetilde{u}(\eta) = \sqrt{w^2 \cdot \left(h_1^2(\eta/w + x_2 x_3) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3)\right) + \eta^2 u_\text{rel}^2(w)} \ . \tag{12}$$

With time preselection and because of $x_1 = n_\text{g}/t_\text{g}$ and $x_2 = r_0$,

$$n_\text{g} = t_\text{g} \cdot (\eta/w + r_0 x_3) \tag{13}$$

is obtained from equation (11). Then, with $h_1^2(x_1) = x_1/t_\text{g} = n_\text{g}/t_\text{g}^2$ and by substituting n_g according to equation (13) and with $u^2(x_2) = r_0/t_0$, equation (12) leads to

$$\widetilde{u}(\eta) = \sqrt{w^2 \cdot \left((\eta/w + r_0 x_3)/t_\text{g} + x_3^2 r_0/t_0 + r_0^2 u^2(x_3)\right) + \eta^2 u_\text{rel}^2(w)} \ . \tag{14}$$

With preselection of counts,

$$t_\text{g} = \frac{n_\text{g}}{\eta/w + r_0 x_3} \tag{15}$$

is analogously obtained. Then, with $h_1^2(x_1) = x_1^2/n_\text{g} = n_\text{g}/t_\text{g}^2$ and by substituting t_g according to equation (15) and again with $u^2(x_2) = r_0/t_0$, equation (12) leads to

$$\widetilde{u}(\eta) = \sqrt{w^2 \cdot \left((\eta/w + r_0 x_3)^2/n_\text{g} + x_3^2 r_0/t_0 + r_0^2 u^2(x_3)\right) + \eta^2 u_\text{rel}^2(w)} \ . \tag{16}$$

Equation (22) has a solution, the detection limit η^*, if with time preselection the condition

$$k_{1-\beta}\, u_\text{rel}(w) < 1 \tag{17}$$

or with preselection of counts the condition

$$k_{1-\beta} \cdot \sqrt{\frac{1}{n_\text{g}} + u_\text{rel}^2(w)} < 1 \tag{18}$$

is fulfilled. Otherwise, it can happen that a detection limit does not exist because of too great an uncertainty of the quantities X_4 to X_m, summarily expressed by $u_\text{rel}(w)$. The condition according to equation (17) also applies in the case of equation (12), if $h_1(x_1)$ increases for growing x_1 more slowly than x_1, i.e. if $h_1(x_1)/x_1 \to 0$ for $x_1 \to \infty$.

5.3.3 Approximations

It is often sufficient to use the following approximations for the function $\widetilde{u}(\eta)$, in particular, if the standard uncertainty $u(x_1)$ is not known as a function $h_1(x_1)$. A prerequisite is that measurement results y_j and associated standard uncertainties $u(y_j)$, calculated according to 5.1 and 5.2 from some previous measurements of the same kind, are already available (j = 0,1,2,...). The measurements have to be carried out on different samples with differing activities, but in other respects as far as possible under similar conditions. One of the measurements can be a background effect measurement or a blank measurement with $\eta = 0$ and, for instance, $j = 0$. Then, $y_0 = 0$ has to be set and $\widetilde{u}(0) = u(y_0)$. The measurement currently carried out can be taken as a further measurement with $j = 1$.

The function $\widetilde{u}(\eta)$ often shows a rather slow increase. Therefore, the approximation $\widetilde{u}(\eta) = u(y_1)$ is sufficient in some of these cases, especially if the primary measurement result y_1 of the measurand is not much larger than the associated standard uncertainty $u(y_1)$.

If only $\widetilde{u}(0) = u(y_0)$ and $y_1 > 0$ with $u(y_1)$ are known, then the following linear interpolation often suffices:

$$\widetilde{u}^2(\eta) = \widetilde{u}^2(0)\,(1 - \eta/y_1) + u^2(y_1)\,\eta/y_1 \ . \tag{19}$$

If the results y_0, y_1, and y_2 as well as the associated standard uncertainties $u(y_0)$, $u(y_1)$, and $u(y_2)$ from three measurements are available, then the following bilinear interpolation can be used:

$$\widetilde{u}^2(\eta) = u^2(y_0) \cdot \frac{(\eta - y_1)(\eta - y_2)}{(y_0 - y_1)(y_0 - y_2)} + u^2(y_1) \cdot \frac{(\eta - y_0)(\eta - y_2)}{(y_1 - y_0)(y_1 - y_2)} + u^2(y_2) \cdot \frac{(\eta - y_0)(\eta - y_1)}{(y_2 - y_0)(y_2 - y_1)} \ . \tag{20}$$

If results from many similar measurements are given, then the parabolic shape of the function $\widetilde{u}^2(\eta)$ can also be determined by an adjustment calculation.

6 Characteristic limits and assessments

6.1 Specifications

The probability α of the error of the first kind, the probability β of the error of the second kind, and the probability $1-\gamma$ for the confidence interval must be specified. The choice $\alpha = \beta$ and the value 0,05 for α, β, and γ are recommended. Then, $k_{1-\alpha} = k_{1-\beta} =$ 1,65 and $k_{1-\gamma/2} =$ 1,96 (see Annex E).

If it is to be assessed whether or not a measurement procedure for the measurand satisfies the requirements to be fulfilled for scientific, legal or other reasons (see 6.6), then a guideline value η_r as a value of the measurand, for instance, an activity, must also be specified.

6.2 Decision threshold

The decision threshold y^* of the non-negative measurand according to 5.1, which quantifies the physical effect of interest, is that value of the estimator Y which, if exceeded by a determined value of Y, the primary measurement result y, allows the conclusion that the physical effect is present. Otherwise, this effect is assumed to be absent. If the physical effect is really absent, then this decision rule leads at most with the specified probability α to the then wrong decision that the effect is present (error of the first kind; see 6.1 and 6.5).

A determined primary measurement result y for the non-negative measurand is only significant for the true value of the measurand to differ from zero ($\eta > 0$), if it is unlikely enough on the hypothesis of $\eta = 0$. The primary measurement result y must therefore be larger than the decision threshold

$$y^* = k_{1-\alpha}\widetilde{u}(0) \ . \tag{21}$$

With the approximation $\widetilde{u}(\eta) = u(y)$ (see 5.3.3), $y^* = k_{1-\alpha}u(y)$ applies.

6.3 Detection limit

The detection limit η^* is the smallest true value of the measurand, for which, by applying the decision rule according to 6.2, the probability of the wrong assumption that the physical effect is absent (error of the second kind) does not exceed the specified probability β (see 6.1).

In order to find out whether a measurement procedure is suitable for the measurement purpose, the detection limit η^* is compared with the specified guideline value η_r of the measurand (see 6.1 and 6.6). The detection limit η^* is the smallest true value of the measurand which can be detected with the measurement procedure to be applied. It is so high above the decision threshold y^* that the probability of the error of the second kind does not exceed β. The detection limit is provided as the smallest solution of the equation

$$\eta^* = y^* + k_{1-\beta}\widetilde{u}(\eta^*) \ . \tag{22}$$

$\eta^* \geq y^*$ always applies. Equation (22) is an implicit equation, its right-hand side also depends on η^*. The detection limit can be calculated by solving equation (22) for η^* or, more simply, however, by iteration: repeatedly substituting an approximation η_i for η^* in the right-hand side of equation (22) produces an improved approximation η_{i+1} according to (see Figure 1):

$$\eta_{i+1} = y^* + k_{1-\beta}\widetilde{u}(\eta_i) \ . \tag{23}$$

As a starting approximation, for instance, $\eta_0 = 2y^*$ can be chosen. The iteration converges in most cases, but not, if equation (22) does not have a solution η^*. In the latter case or if $\eta^* < y^*$ results, the detection limit does not exist (see 6.6).

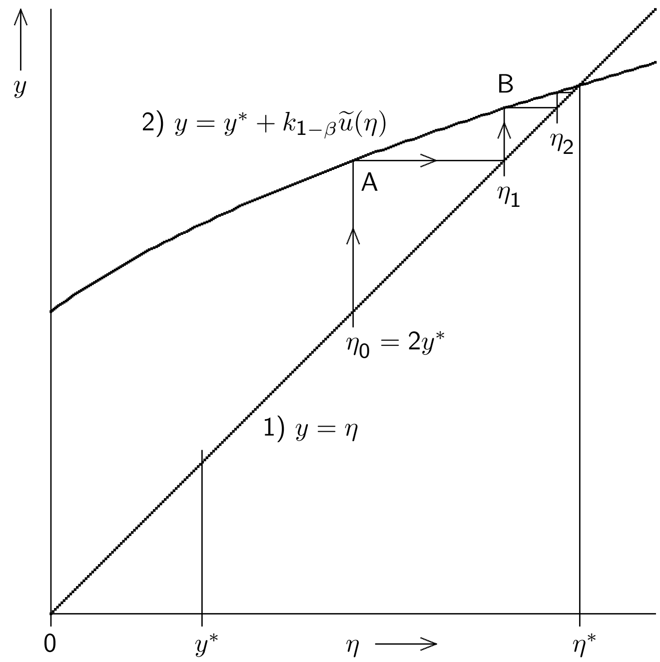

Figure 1: Calculation of the detection limit by iteration

With the iteration according to equations (23) or (24) and beginning with a starting approximation η_0, for instance, $\eta_0 = 2y^*$ as shown, the sequences of the improved approximations η_i $(i = 1,2,\ldots)$ converge to the detection limit η^*, which is the abscissa of the intersection point of straight line 1 and curve 2. y^* is the decision threshold. With the alternative application of the regula falsi according to equation (24), the sequence η_i is generated by means of secants of curve 2, for instance, through points A and B. The shown hyperbolic shape of curve 2 is typical of many applications, for instance, those with equations (14) or (16). The detection limit does not exist if curve 2 does not intersect straight line 1 at any abscissa $\eta \geq y^*$.

After the calculation of η_1 or, for instance, with the choice of $\eta_1 = 3y^*$, it is more advantageous for $i \geq 1$ to apply the regula falsi, which in general converges more rapidly. For this purpose, equation (23) has to be replaced by

$$\eta_{i+1} = \frac{y^* + k_{1-\beta} \cdot (\eta_i \widetilde{u}(\eta_j) - \eta_j \widetilde{u}(\eta_i))/(\eta_i - \eta_j)}{1 - k_{1-\beta} \cdot (\widetilde{u}(\eta_i) - \widetilde{u}(\eta_j))/(\eta_i - \eta_j)} \tag{24}$$

with $j < i$. Then, $j = 0$ should be set or j be fixed after several iteration steps.

Any iteration must be stopped if a specified accuracy of ν digits is attained, i.e. if the ν first digits of the successive approximations no longer change. But if a too high accuracy is demanded, then, even with an iteration converging in principle, the successive approximations in general permanently fluctuate around and close to the exact solution but never attain it. A smaller ν must then be chosen.

With the approximation $\widetilde{u}(\eta) = u(y)$ (see 5.3.3), $\eta^* = (k_{1-\alpha} + k_{1-\beta})\, u(y)$ applies.

The linear interpolation according to equation (19) leads to the approximation

$$\eta^* = a + \sqrt{a^2 + (k_{1-\beta}^2 - k_{1-\alpha}^2)\, \widetilde{u}^2(0)} \ ; \quad a = k_{1-\alpha}\widetilde{u}(0) + \tfrac{1}{2}(k_{1-\beta}^2/y_1)(u^2(y_1) - \widetilde{u}^2(0)) \ . \tag{25}$$

If $\alpha = \beta$, then $\eta^* = 2a$ follows.

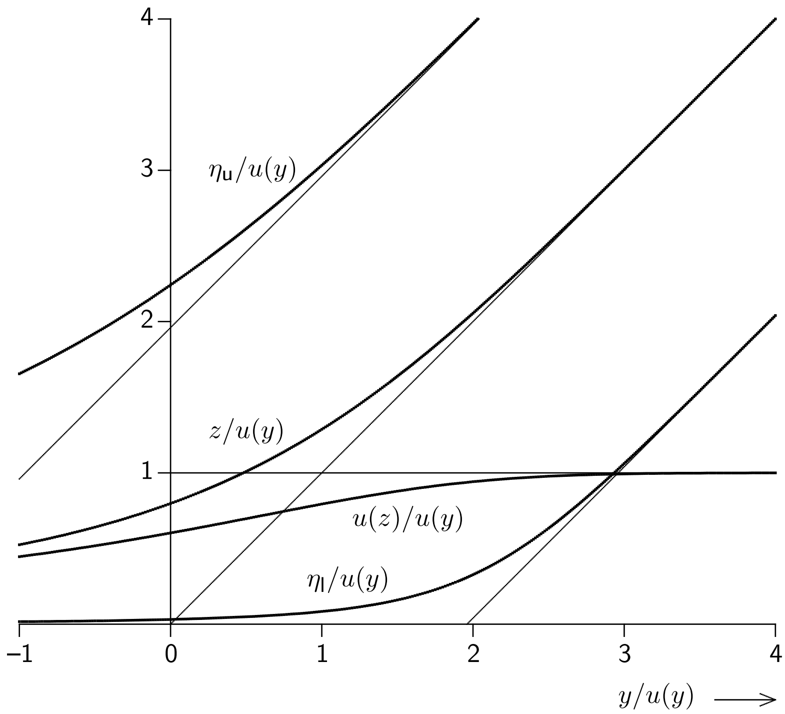

Figure 2: Best estimate and confidence limits

Best estimate z of the measurand, associated standard uncertainty $u(z)$, lower confidence limit η_l and upper confidence limit η_u as functions of the primary measurement result y. All these values are scaled with the standard uncertainty $u(y)$ and $\gamma = 0{,}05$ is chosen. The ascending straight lines and the horizontal straight line with ordinate 1 are asymptotes. The relations $0 < \eta_l < z < \eta_u$ and $z > y$ as well as $u(z) < u(y)$ and $u(z) < z$ apply, and moreover $\eta_l > y - k_{1-\gamma/2}\, u(y)$ and $\eta_u > y + k_{1-\gamma/2}\, u(y)$.

6.4 Confidence limits

The confidence limits as limits of a confidence interval are provided for a physical effect, recognized as present according to 6.2, in such a way that the confidence interval contains the true value of the measurand with the specified probability $1-\gamma$ (see 6.1). The confidence limits take into account that the measurand is non-negative.

With a present primary measurement result y of the measurand and the standard uncertainty $u(y)$ associated with y (see 5.2), the lower confidence limit η_l and the upper confidence limit η_u are provided by

$$\eta_l = y - k_p\, u(y) \;\; ; \quad p = \omega \cdot (1 - \gamma/2) \;\; ; \tag{26}$$

$$\eta_u = y + k_q\, u(y) \;\; ; \quad q = 1 - \omega\gamma/2 \tag{27}$$

where

$$\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y/u(y)} \exp(-v^2/2)\, \mathrm{d}v = \Phi(y/u(y)) \;\; . \tag{28}$$

For the distribution function $\Phi(t)$ of the standardized normal distribution and for its inversion $k_p = t$ for $\Phi(t) = p$, see Table E.1. For methods for its calculation, see Annex E or, for instance, [8] or [9].

In general, the confidence limits are located neither symmetrical to y nor to the best estimate z (see 6.5 and Figure 2), but the probabilities of the measurand being smaller than η_l or larger than η_u both equal $\gamma/2$. The relations $0 < \eta_l < \eta_u$ apply.

$\omega = 1$ may be set if $y \geq 4u(y)$. In this case, the following approximations symmetrical to y apply:

$$\eta_{u,l} = y \pm k_{1-\gamma/2}\, u(y) \;\; . \tag{29}$$

6.5 Assessment of a measurement result

The determined primary measurement result y of the measurand must be compared with the decision threshold y^*. If $y > y^*$, then the physical effect quantified by the measurand is recognized as present. Otherwise, the hypothesis that the effect is absent cannot be rejected.

If $y > y^*$ and with ω according to equation (28), the best estimate z of the measurand is given by (see 5.1 NOTE and Figure 2)

$$z = y + \frac{u(y)\ \exp\big(-y^2/(2u^2(y))\big)}{\omega\,\sqrt{2\pi}} \ . \tag{30}$$

The standard uncertainty associated with z reads

$$u(z) = \sqrt{u^2(y) - (z-y)z} \ . \tag{31}$$

The relations $z > y$ and $z > 0$ and $\eta_l < z < \eta_u$ as well as $u(z) < u(y)$ and $u(z) < z$ apply, moreover, for $y \geq 4u(y)$, the approximations

$$z = y \ ; \quad u(z) = u(y) \ . \tag{32}$$

6.6 Assessment of a measurement procedure

The decision on whether or not a measurement procedure to be applied sufficiently satisfies the requirements regarding the detection of the physical effect quantified by the measurand is made by comparing the detection limit η^* with the specified guideline value η_r. If $\eta^* > \eta_r$ or if equation (22) has no solution η^*, then the measurement procedure is not suitable for the intended measurement purpose with respect to the requirements.

To improve the situation in the case of $\eta^* > \eta_r$, it can often be sufficient to choose longer measurement durations or to preselect more counts of the measurement procedure. This reduces the detection limit.

7 Documentation

After the determination of the characteristic limits, a report containing the following information must be prepared:

a) test laboratory;

b) reference to the determination according to the present standard proposal on the basis of DIN 25482-10 or ISO 11929-7;

c) physical effect of interest, measurand, and model of the evaluation;

d) probabilities α and β of the errors of the first and second kind, respectively, and, if necessary, guideline value η_r;

e) primary measurement result y and standard uncertainty $u(y)$ associated with y;

f) decision threshold y^*;

g) detection limit η^*;

h) if necessary, statement whether or not the measurement procedure is suitable for the intended measurement purpose;

i) statement whether or not the physical effect is recognized as present;

j) in addition, if the physical effect is recognized as present, lower confidence limit η_l and upper confidence limit η_u with the probability $1-\gamma$ for the confidence interval, best estimate z of the measurand, and standard uncertainty $u(z)$ associated with z;

k) if necessary, deviations from the present standard proposal;

l) testing person, test location, test date, and signature.

Annex A

(normative)

Overview of the general procedure

A.1 Introduction of the model

Introduction of the non-negative measurand Y and of its representation as a function of the input quantities X_i (model; X_1 is the gross effect; see 5.1 and 5.2.1):

$$Y = G(X_1, \ldots, X_m) \ . \tag{A.1}$$

A.2 Preparation of the input data and specifications

Determination of the estimates x_i of the input quantities X_i with the associated standard uncertainties $u(x_i)$ according to DIN 1319-3, DIN 1319-4, DIN V ENV 13005, [1], or [3] from measurements and previous investigations. For a count rate $X_i = \varrho_i$ with the counting result n_i obtained from a measurement of duration t_i, introduce $x_i = n_i/t_i$ and $u^2(x_i) = n_i/t_i^2$ (see 5.2.1). In particular, $u(x_1) = h_1(x_1) = \sqrt{x_1/t_1}$ then applies for the gross effect X_1 (see 5.3.2 and A.4).

Specifications: probabilities α, β and γ and the guideline value η_{r} (see 6.1).

A.3 Calculation of the primary measurement result y with the associated standard uncertainty $u(y)$

$$y = G(x_1, \ldots, x_m) \ ; \tag{A.2}$$

$$u^2(y) = \sum_{i=1}^{m} \left(\frac{\partial G}{\partial X_i}\right)^2 u^2(x_i) \tag{A.3}$$

for presupposed uncorrelated input quantities X_i (see 5.2.1 and A.2). Otherwise, see the references in A.2. The estimates $x_1, \ldots, x_m$ must be substituted in $\partial G/\partial X_i$.

A.4 Calculation of the standard uncertainty $\widetilde{u}(\eta)$

If $u(x_1)$ is known as a function $h_1(x_1)$, y is replaced by η and equation (A.2) is solved for x_1. With η specified, x_1 can also be numerically calculated from equation (A.2), for instance, by means of an iteration procedure. This results in x_1 as a function of η and $x_2, \ldots, x_m$. The function replaces x_1 in equation (A.3) and in $h_1(x_1)$. This yields $\widetilde{u}(\eta)$ instead of $u(y)$ (see 5.3.2). Otherwise, $\widetilde{u}(\eta)$ follows as an approximation by interpolating the data y and $u(y)$ from several measurements (see 5.3.3).

A.5 Calculation of the decision threshold y^*

$$y^* = k_{1-\alpha}\widetilde{u}(0) \tag{A.4}$$

(see 6.2). Assessment: an effect of the measurand Y is recognized as present if $y > y^*$ (see 6.5). If not, A.7 and A.8 are omitted.

A.6 Calculation of the detection limit η^*

The detection limit η^* is the smallest solution of the equation

$$\eta^* = y^* + k_{1-\beta}\widetilde{u}(\eta^*) \ . \tag{A.5}$$

It can be calculated by iteration with the starting approximation $\eta^* = 2y^*$ (see 6.3). Assessment: the measurement procedure is not suitable for the measurement purpose if $\eta^* > \eta_{\mathrm{r}}$ or if η^* does not exist (see 6.6).

A.7 Calculation of the confidence limits η_{l} and η_{u}

$$\eta_{\mathrm{l}} = y - k_p\, u(y) \ \text{ with } \ p = \omega \cdot (1 - \gamma/2) \ ; \quad \eta_{\mathrm{u}} = y + k_q\, u(y) \ \text{ with } \ q = 1 - \omega\gamma/2 \tag{A.6}$$

where $\omega = \Phi(y/u(y))$ (see 6.4; for the calculation of ω, k_p, and k_q, see Annex E).

A.8 Calculation of the best estimate z of the measurand with the associated standard uncertainty $u(z)$

$$z = y + \frac{u(y)\, \exp\big(-y^2/(2u^2(y))\big)}{\omega\sqrt{2\pi}} \ ; \quad u(z) = \sqrt{u^2(y) - (z-y)z} \tag{A.7}$$

(see 6.5).

A.9 Preparation of the documentation

Report of the results of A.1 to A.8 (see Section 7).

Annex B
(normative)
Various applications

B.1 General aspects

The procedure described in the main part of this standard proposal is so general that it allows a large variety of applications to similar measurements. Some important cases are treated in the following. They do not differ in their models from those in the main part, but merely in the interpretation of the input quantities X_1 and X_2 and in setting up the corresponding estimates x_1 and x_2 and standard uncertainties $u(x_1)$ and $u(x_2)$.

With each of the following applications dealt with in Annexes B and C, the respective main task consists in determining the primary measurement result y of the measurand and the associated standard uncertainty $u(y)$ according to 5.2 or A.3 as well as the standard uncertainty $\widetilde{u}(\eta)$ as a function of the measurand according to 5.3 or A.4. Subsequently, with all applications, the decision threshold y^*, the detection limit η^*, the confidence limits η_l and η_u, and the best estimate z of the measurand with the associated standard uncertainty $u(z)$ have to be calculated in the same way according to Section 6 or A.5 to A.8. This is no longer pointed out in the following. Numerical examples of the applications are treated in Annex D.

B.2 Counting measurements on moving objects

The application of this standard proposal to counting measurements on moving objects is also treated in DIN 25482-13 and ISO 11929-6. During such a measurement, the measurement object is moved along a specified measurement distance on a straight line passing an ionizing-radiation detector (or vice versa). Data obtained from the measurement during this travel are, on the one hand, the counted numbers n_g or n_0 of the recorded pulses and, on the other hand, the measurement durations t_g or t_0, respectively. In general, the measurement durations can be determined with measurement uncertainties negligible compared to all other measurement uncertainties that must be taken into account. Therefore, they can be taken as constants and the measurement as a measurement with time preselection.

The reduction of the background count rate by the shielding effect of the measurement object can be taken into account by means of the shielding factor f by setting $X_3 = f$ in equation (4). f can be obtained experimentally from previous measurements as an arithmetic mean value and the standard uncertainty $u(f)$ associated with f as the empirical standard deviation of the arithmetic mean value. They can alternatively be obtained as the expectation value and the standard deviation $u(f) = \Delta f/\sqrt{12}$, respectively, from a rectangular distribution with the width Δf over the region of the possible values of f.

In the simplest case where the model has to be specified in the form of $Y = X_1 - X_2 X_3 = \varrho_\mathrm{g} - \varrho_0 f$ and where the measurement durations t_g and t_0 are preselected and the estimates $x_1 = n_\mathrm{g}/t_\mathrm{g} = r_\mathrm{g}$ and $x_2 = n_0/t_0 = r_0$ with the associated squared standard uncertainties $u^2(x_1) = r_\mathrm{g}/t_\mathrm{g}$ and $u^2(x_2) = r_0/t_0$ are applied, the results read

$$y = \frac{n_\mathrm{g}}{t_\mathrm{g}} - \frac{n_0}{t_0} f = r_\mathrm{g} - r_0 f \ ; \quad u(y) = \sqrt{\frac{r_\mathrm{g}}{t_\mathrm{g}} + \frac{r_0}{t_0} f^2 + r_0^2 u^2(f)} \ . \tag{B.1}$$

Replacing y by η and eliminating $r_\mathrm{g} = \eta + r_0 f$, because of $u^2(x_1) = h_1^2(x_1) = x_1/t_\mathrm{g} = r_\mathrm{g}/t_\mathrm{g}$, yields

$$\widetilde{u}(\eta) = \sqrt{\frac{\eta + r_0 f}{t_\mathrm{g}} + \frac{r_0}{t_0} f^2 + r_0^2 u^2(f)} \ . \tag{B.2}$$

B.3 Measurements with ratemeters

A ratemeter is here understood as a linear, analogously working count rate measuring instrument where the output signal increases sharply (with a negligible rise time constant) upon the arrival of an input pulse and then decreases exponentially with a relaxation time constant τ until the next input pulse arrives. The signal increase must be the same for all pulses and the relaxation time constant must be independent of the count rate. A digitally working count rate measuring instrument simulating the one just described is also taken as a ratemeter that has to be considered here.

Each particular measurement using a ratemeter must be carried out in the stationary state of the ratemeter. This requires at least a sufficiently fixed time span between the start of measurement and reading the ratemeter indication. This applies to each sample and to each background effect measurement. According to [10], fixed time spans of 3τ

or 7τ correspond to deviations of the indication by 5 % or 0,1 % of the magnitude of the difference between the indication at the start of measurement and that at the end of the time span. If further uncertain influences have to be taken into account, then a time span of 7τ should be chosen, if possible.

The expectation values ϱ_g and ϱ_0 of the output signals of the ratemeter in the cases of measuring the gross and background effects, respectively, are taken as the input quantities X_1 and X_2 for the calculation of the characteristic limits: $X_1 = \varrho_\text{g}$ and $X_2 = \varrho_0$. With the values r_g and r_0 of the output signals determined at the respective moments of measurement, the following approaches result for the values of the input quantities and the associated standard uncertainties:

$$x_1 = r_\text{g} \ ; \quad x_2 = r_0 \ ; \tag{B.3}$$

$$u^2(x_1) = \frac{r_\text{g}}{2\tau_\text{g}} \ ; \quad u^2(x_2) = \frac{r_0}{2\tau_0} \ . \tag{B.4}$$

In equation (B.4), approximations with a maximum relative deviation of 5 % for $r_\text{g}\tau_\text{g} \geq 0{,}65$ and of 1 % for $r_\text{g}\tau_\text{g} \geq 1{,}32$ are specified according to [10]. The same applies to $r_0\tau_0$. The relaxation time constants τ_g and τ_0 have to be adjusted accordingly.

The ratemeter measurement is equivalent to a counting measurement with time preselection according to 5.3.2 and with the measurement durations $t_\text{g} = 2\tau_\text{g}$ and $t_0 = 2\tau_0$. The quotients n_g/t_g and n_0/t_0 of the counting measurement have to be replaced here by the measured count rate values r_g and r_0, respectively, of the ratemeter measurement. This applies, in particular, to equation (13). See also the numerical example in D.2.2. The standard uncertainties of the relaxation time constants do not appear in the equations and are therefore not needed.

In the simplest case where the model has to be specified in the form of $Y = X_1 - X_2 = \varrho_\text{g} - \varrho_0$, equations (B.3) and (B.4) lead to

$$y = r_\text{g} - r_0 \ ; \quad u(y) = \sqrt{\frac{r_\text{g}}{2\tau_\text{g}} + \frac{r_0}{2\tau_0}} \ . \tag{B.5}$$

Replacing y by η and eliminating $r_\text{g} = \eta + r_0$, because of $u^2(x_1) = h_1^2(x_1) = x_1/(2\tau_\text{g}) = r_\text{g}/(2\tau_\text{g})$, yields

$$\widetilde{u}(\eta) = \sqrt{\frac{\eta + r_0}{2\tau_\text{g}} + \frac{r_0}{2\tau_0}} \ . \tag{B.6}$$

B.4 Repeated counting measurements with random influences

B.4.1 General aspects

Random influences due to, for instance, sample treatment and instruments cause measurement deviations, which can be different from sample to sample. In such cases, the counting results n_i of the counting measurements on several samples of a radioactive material to be examined, on several blanks of a radioactively labelled blank material, and on several reference samples of a standard reference material are therefore respectively averaged to obtain suitable estimates x_1 and x_2 of the input quantities X_1 and X_2 and the associated standard uncertainties $u(x_1)$ and $u(x_2)$, respectively. Accordingly, X_1 has to be considered as the mean gross count rate and X_2 as the mean background count rate. Therefore, the measurand Y with the wanted true value η has also to be taken as an averaged quantity, for instance, as the mean net count rate or mean activity of the samples. In the following, the symbols belonging to the countings on the samples, blanks, and reference samples are marked by the subscripts b, 0, and r, respectively. In each case, arithmetic averaging over m countings of the same kind carried out with the same preselected measurement duration t (time preselection) is denoted by an overline. For $m > 1$ counting results n_i which are obtained in such a way and have to be averaged, the mean value $\overline{n}$ and the empirical variance s^2 of the values n_i are given by

$$\overline{n} = \frac{1}{m}\sum_{i=1}^{m} n_i \ ; \quad s^2 = \frac{1}{m-1}\sum_{i=1}^{m}(n_i - \overline{n})^2 \ . \tag{B.7}$$

The following procedures are approximations for sufficiently large counting results n_i and $\overline{n} \gg s/\sqrt{m}$, which allow the random influences to be recognized in addition to those of the Poisson statistics (see also under equation (B.12)).

For the calculation of the characteristic limits, not only t_g, but also m_g must be specified.

A numerical example of a measurement with random influences is described in D.3.

B.4.2 Procedure with unknown influences

In the case of unknown influences, the following expressions are valid for the mean gross count rate X_1 and the mean background count rate X_2:

$$x_1 = \overline{n}_\mathrm{g}/t_\mathrm{g} \ ; \quad x_2 = \overline{n}_0/t_0 \ ; \tag{B.8}$$

$$u^2(x_1) = s_\mathrm{g}^2/(m_\mathrm{g} t_\mathrm{g}^2) \ ; \quad u^2(x_2) = s_0^2/(m_0 t_0^2) \ . \tag{B.9}$$

With the approaches according to equations (B.8) and (B.9), equations (6) and (9) yield

$$y = \left(\frac{\overline{n}_\mathrm{g}}{t_\mathrm{g}} - \frac{\overline{n}_0}{t_0} x_3 \right) \cdot w \ ; \tag{B.10}$$

$$u(y) = \sqrt{w^2 \cdot \left(s_\mathrm{g}^2/(m_\mathrm{g} t_\mathrm{g}^2) + x_3^2 s_0^2/(m_0 t_0^2) + (\overline{n}_0/t_0) u^2(x_3) \right) + y^2 u_\mathrm{rel}^2(w)} \ . \tag{B.11}$$

$u^2(x_1)$ is not given as a function $h_1^2(x_1)$ of x_1. Therefore, $\widetilde{u}^2(\eta)$ must be determined as an approximation according to 5.3.3, for instance, according to equation (19), where the current result y can be used as y_1. For this purpose and for the calculation of $\widetilde{u}^2(0)$, i.e. for $\eta = 0$, the variance s_g^2 has to be replaced by s_0^2.

B.4.3 Procedure with known influences

Another procedure, appropriate when small random influences are present, is based on the approach

$$s^2 = \overline{n} + \vartheta^2 \overline{n}^2 \ . \tag{B.12}$$

The term linear in $\overline{n}$ of equation (B.12) follows from the Poisson distributions of the numbers N_i of pulses when random influences disappear. These influences are described by the term square in $\overline{n}$ assuming an empirical relative standard deviation ϑ valid for all samples and countings and caused by these influences. This influence parameter ϑ can be calculated from countings on reference samples according to equation (B.12) by equating with equation (B.7):

$$\vartheta^2 = (s_\mathrm{r}^2 - \overline{n}_\mathrm{r})/\overline{n}_\mathrm{r}^2 \ . \tag{B.13}$$

Instead of the data from countings on reference samples, those on other samples can be used which were previously examined, not explicitly for reference purposes but under conditions similar to those of the reference samples.

If $\vartheta^2 < 0$ results, the approach and the data are not compatible. The number m_r of the reference samples should then be enlarged or $\vartheta = 0$ be set. Moreover, $\vartheta < 0{,}2$ should be obtained. Otherwise, one can proceed according to B.4.2.

Instead of equation (B.9), the expressions

$$u^2(x_1) = (\overline{n}_\mathrm{g} + \vartheta^2 \overline{n}_\mathrm{g}^2)/(m_\mathrm{g} t_\mathrm{g}^2) \ ; \quad u^2(x_2) = (\overline{n}_0 + \vartheta^2 \overline{n}_0^2)/(m_0 t_0^2) \tag{B.14}$$

now apply with equation (B.12). The cases $m_\mathrm{g} = 1$ and $m_0 = 1$ are permitted here. Therefore, with $x_1 = \overline{n}_\mathrm{g}/t_\mathrm{g}$ and equation (B.14), $u^2(x_1)$ is given as a function of x_1 by

$$u^2(x_1) = h_1^2(x_1) = (x_1/t_\mathrm{g} + \vartheta^2 x_1^2)/m_\mathrm{g} \ . \tag{B.15}$$

Equations (B.8) and (B.10) remain valid. Furthermore, according to equation (9) with equations (B.8) and (B.14), it follows that

$$u(y) = \sqrt{w^2 \cdot \left(u^2(x_1) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3) \right) + y^2 u_\mathrm{rel}^2(w)} \ . \tag{B.16}$$

In order to calculate $\widetilde{u}(\eta)$, the result y is replaced by η and equation (B.10) is solved for $x_1 = \overline{n}_\mathrm{g}/t_\mathrm{g}$. This yields $x_1 = \eta/w + \overline{n}_0 x_3/t_0$. The estimate x_1, determined in this way in the current case, has to be substituted in equation (B.15) and $u^2(x_1)$ obtained therefrom in equation (B.16). This finally leads to $\widetilde{u}(\eta)$ (see also 5.3).

The condition according to equation (17) has to be replaced here by the condition

$$k_{1-\beta} \cdot \sqrt{\frac{\vartheta^2}{m_\mathrm{g}} + u_\mathrm{rel}^2(w)} < 1 \ . \tag{B.17}$$

B.5 Counting measurements on filters during accumulation of radioactive material

B.5.1 General aspects

For monitoring flowing fluid media (gas or liquid, for instance, vent air or room air in nuclear installations or water), a counting measurement is continuously carried out on a filter during the accumulation of radioactive material from the medium. The application of this standard proposal to such a measurement is also treated in ISO 11929-5. The measurement consists in a temporal sequence of consecutive measurement intervals of the same duration t. The half-lives of the nuclides accumulated on the filter are assumed to be long compared to the total duration of all measurement intervals, the data of which are used in the following calculation of the characteristic limits. In addition, the background effect is assumed to remain constant during the whole measurement. There are two measurands Y of interest:

a) the activity concentration $A_{V,j}$ (activity divided by the total volume of the sample, see ISO 31-9) of the radioactive nuclides entrained by the medium, accumulated on the filter, and measured during the measurement interval j of duration t (case a, see B.5.2) and
b) the change $\Delta A_{V,j}$ in the activity concentration according to case a, compared with the mean activity concentration $\overline{A}_{V,j}$ from m preceding measurement intervals (case b, see B.5.3).

It is sufficient for cases a and b to introduce the respective models according to 5.2 that describe the measurands $Y = A_{V,j}$ and $Y = \Delta A_{V,j}$ as functions of the input quantities X_i and to specify the estimates x_i with the associated standard uncertainties $u(x_i)$ of the input quantities X_i. Everything else then follows according to 5.2.2, 5.3.2 and Section 6 and analogously to B.2 and B.3. A numerical example is described in D.4.

The activity is divided by the sample volume, i.e. by the volume V of the medium flowing through the filter during the measurement of duration t. This volume V with the associated standard uncertainty $u(V)$ as well as a calibration factor ε, which has to be considered with the associated standard uncertainty $u(\varepsilon)$, are assumed to be known from previous investigations. The efficiency of the filter is assumed to be contained in ε. The standard uncertainty $u(t)$ of the measurement duration t is neglected since t can be measured by far more exactly than all the other quantities involved and can thus be taken as a constant.

B.5.2 Activity concentration as the measurand

In case a, $Y = A_{V,j}$ is the measurand of the measurement interval j. The input quantities X_i are specified as follows: $X_1 = \varrho_j$, $X_2 = \varrho_{j-1}$, $X_5 = \varepsilon$, and $X_7 = V$, where ϱ_j is the gross count rate in the measurement interval j. There are no further input quantities, they are set constant equalling 1. The model according to equation (4) now reads

$$Y = A_{V,j} = \frac{X_1 - X_2}{X_5 X_7} = \frac{\varrho_j - \varrho_{j-1}}{\varepsilon V} \quad . \tag{B.18}$$

Because of the background effect assumed to be constant, its contributions cancel out in the difference.

Similar to 5.2.2, the estimates x_1 and x_2 with the associated standard uncertainties $u(x_1)$ and $u(x_2)$ of the input quantities X_1 and X_2, respectively, are specified as follows with n_j being the number of events recorded in the measurement interval j:

$$x_1 = r_j = n_j/t \ ; \qquad u^2(x_1) = r_j/t \ ; \tag{B.19}$$

$$x_2 = r_{j-1} = n_{j-1}/t \ ; \quad u^2(x_2) = r_{j-1}/t \ . \tag{B.20}$$

Obviously, $u(x_1)$ is thus known as a function $h_1(x_1)$ of x_1, which is needed for the decision threshold and the detection limit, since

$$u(x_1) = \sqrt{r_j/t} = h_1(x_1) = \sqrt{x_1/t} \ . \tag{B.21}$$

With the preceding approaches and $x_3 = 1$ with $u(x_3) = 0$ as well as $w = 1/(\varepsilon V)$ with $u_{\mathrm{rel}}^2(w) = u^2(\varepsilon)/\varepsilon^2 + u^2(V)/V^2$, the following is obtained according to 5.2.2 and 5.3.2:

$$y = \frac{x_1 - x_2}{x_5 x_7} = \frac{r_j - r_{j-1}}{\varepsilon V} \ ; \tag{B.22}$$

$$\begin{aligned} u(y) &= \sqrt{w^2 \cdot \left(u^2(x_1) + u^2(x_2)\right) + y^2 u_{\mathrm{rel}}^2(w)} \\ &= \frac{1}{\varepsilon V}\sqrt{\frac{r_j + r_{j-1}}{t} + (r_j - r_{j-1})^2 \Big(\frac{u^2(\varepsilon)}{\varepsilon^2} + \frac{u^2(V)}{V^2}\Big)} \ . \end{aligned} \tag{B.23}$$

Replacing y by η yields with equations (B.23) and (12)

$$x_1 = r_j = \eta/w + x_2 = \eta\,\varepsilon\,V + r_{j-1} \quad ; \tag{B.24}$$

$$\begin{aligned}\widetilde{u}(\eta) &= \sqrt{w^2 \cdot \big(h_1^2(\eta/w + x_2) + u^2(x_2)\big) + \eta^2 u_{\text{rel}}^2(w)} \\ &= \sqrt{\frac{(\eta\,\varepsilon\,V + x_2)/t + u^2(x_2)}{(\varepsilon\,V)^2} + \eta^2 \cdot \Big(\frac{u^2(\varepsilon)}{\varepsilon^2} + \frac{u^2(V)}{V^2}\Big)} \\ &= \sqrt{\frac{\eta\,\varepsilon\,V + 2r_{j-1}}{(\varepsilon\,V)^2\,t} + \eta^2 \cdot \Big(\frac{u^2(\varepsilon)}{\varepsilon^2} + \frac{u^2(V)}{V^2}\Big)} \quad .\end{aligned} \tag{B.25}$$

B.5.3 Change in the activity concentration as the measurand

Case b only differs from case a treated in B.5.2 by a different definition of X_2. The model reads

$$\begin{aligned}Y = \Delta A_{V,j} &= A_{V,j} - \overline{A}_{V,j} = \frac{X_1 - X_2}{X_5 X_7} \\ &= \frac{1}{\varepsilon\,V}\Big(\varrho_j - \varrho_{j-1} - \frac{1}{m}\sum_{k=1}^{m}(\varrho_{j-k} - \varrho_{j-k-1})\Big) = \frac{1}{\varepsilon\,V}\Big(\varrho_j - \Big(1 + \frac{1}{m}\Big)\varrho_{j-1} + \frac{1}{m}\varrho_{j-m-1}\Big) \quad .\end{aligned} \tag{B.26}$$

Instead of $X_2 = \varrho_{j-1}$, now

$$X_2 = \Big(1 + \frac{1}{m}\Big)\varrho_{j-1} - \frac{1}{m}\varrho_{j-m-1} \tag{B.27}$$

is valid with $X_1 = \varrho_j$. Hence follows

$$x_2 = \Big(1 + \frac{1}{m}\Big)r_{j-1} - \frac{1}{m}r_{j-m-1} \quad ; \quad u^2(x_2) = \Big(1 + \frac{1}{m}\Big)^2 \frac{r_{j-1}}{t} + \frac{r_{j-m-1}}{m^2\,t} \quad . \tag{B.28}$$

The values x_2 and $u^2(x_2)$ calculated according to equation (B.28) have to be substituted in equations (B.22) to (B.25) to obtain y, $u(y)$, and $\widetilde{u}(\eta)$.

The count rates of the intermediate intervals $i = j - 2$ to $j - m$ are not involved. They only play a part insofar as with these measurement intervals no measurement effect for $\Delta A_{V,i}$ should be recognized as present, so that a linear increase of the activity on the filter may be assumed.

The model according to equation (B.26) applies to the test for an increase in the activity concentration. If a decrease is to be examined, $Y = \overline{A}_{V,j} - A_{V,j}$ has to be specified as the measurand, i.e. X_1 and X_2 have to be interchanged so that the measurand becomes non-negative as demanded.

Annex C

(normative)

Applications to counting spectrometric measurements

C.1 General aspects

This standard proposal can also be applied to counting spectrometric measurements when a particular line in a measured multi-channel spectrum has to be considered and no adjustment calculations, for instance, an unfolding, have to be carried out. The net intensity of the line is first determined according to C.1 to C.3 by separating the background. Then, if another measurand, for instance, an activity, has to be calculated, one has to proceed according to 5.2 and 5.3 (see C.4).

Independent, Poisson-distributed random variables N_i ($i = 1, \ldots, m$ as well as $i =$ g) are assigned to selected channels of a measured multi-channel spectrum – if necessary, the channels of a channel region of the spectrum can be combined to form a single channel – with the contents n_i of the channels (or channel regions), and the expectation values of the N_i are taken as input quantities X_i (see G.1). In the following, ϑ_i is the lower and ϑ'_i is the upper limit of channel i; ϑ is, for instance, the energy or time or another continuous scaling variable assigned to the channel number. The channel widths $t_i = \vartheta'_i - \vartheta_i$ correspond to t according to G.1. Thus, $X_i = \varrho_i t_i$ with the mean spectral density ϱ_i in channel i, and $x_i = n_i$ is an estimate of X_i with the standard uncertainty $u(x_i) = \sqrt{n_i}$ associated with x_i. For $i = \mathrm{g}$, the quantities N_g and $X_\mathrm{g} = \varrho_\mathrm{g} t_\mathrm{g}$ represent the combined channels of a line of interest in the spectrum. The measurand Y with the true value η is the net intensity of the line, i.e. the expectation value of the net content of channel $i = \mathrm{g}$ (region B, see C.2). (For the appropriate determination of channel regions, see C.3)

At first, the background of the line of interest must be determined, which also includes the contributions of the tails of disturbing lines. A suitable function $H(\vartheta\,; a_1, \ldots, a_m)$, representing the spectral density of the line background with the parameters a_k, is introduced so that

$$n_i = \int_{\vartheta_i}^{\vartheta'_i} H(\vartheta\,; a_1, \ldots, a_m)\,\mathrm{d}\vartheta \;\; ; \quad (i = 1, \ldots, m) \;\; , \tag{C.1}$$

from which the a_k have to be calculated as functions of the n_i.The background contribution to the line is then

$$z_0 = \int_{\vartheta_\mathrm{g}}^{\vartheta'_\mathrm{g}} H(\vartheta\,; a_1, \ldots, a_m)\,\mathrm{d}\vartheta \;\; . \tag{C.2}$$

The random variable Z_0, associated with the background contribution z_0, implicitly is a function of the input quantities X_i because z_0 is calculated from the $x_i = n_i$. The model approach for the measurand Y reads

$$Y = G(X_\mathrm{g}, X_1, \ldots, X_m) = X_\mathrm{g} - Z_0 \tag{C.3}$$

from which

$$y = n_\mathrm{g} - z_0 \;\; ; \quad u^2(y) = n_\mathrm{g} + u^2(z_0) \;\; ; \quad u^2(z_0) = \sum_{i=1}^{m} \Big(\sum_{k=1}^{m} \frac{\partial z_0}{\partial a_k} \frac{\partial a_k}{\partial n_i} \Big)^2 n_i \tag{C.4}$$

follow. The bracketed sum equals $\partial z_0 / \partial n_i$. For the calculation of the function $\widetilde{u}^2(\eta)$, the net content η of channel g is first specified. Then, y in equation (C.4) is replaced by η. This allows n_g to be eliminated, which is not available if η is specified. This results in $n_\mathrm{g} = \eta + z_0$ and

$$\widetilde{u}^2(\eta) = \eta + z_0 + u^2(z_0) \;\; . \tag{C.5}$$

The characteristic limits according to Section 6 then follow from equations (C.4) and (C.5).

If the approach

$$H(\vartheta) = \sum_{k=1}^{m} a_k \psi_k(\vartheta) \tag{C.6}$$

linear in the a_k is applied with given functions $\psi_k(\vartheta)$, then equation (C.1) represents a system of linear equations for the a_k. Thus, the a_k depend linearly on the n_i and the partial derivatives in equation (C.4) do not depend on the n_i. Then,

$$u^2(z_0) = \sum_{i=1}^{m} b_i^2 n_i \tag{C.7}$$

with quantities b_i not depending on the n_i. Equation (C.7) also follows when the background contribution z_0 to the line is calculated linearly from the channel contents n_i with suitably specified coefficients b_i:

$$z_0 = \sum_{i=1}^{m} b_i n_i \;\; . \tag{C.8}$$

C.2 Application according to the background shape

If events of a single line with a known location in the spectrum are to be detected, then the following cases of the background shape as a function of ϑ and the associated approaches have to be distinguished:

a) Constant background: approach $H(\vartheta) = a_1$ (constant, $m = 1$)
b) Linear background, which can often be assumed with gamma radiation: approach $H(\vartheta) = a_1 + a_2\vartheta$ (straight line, $m = 2$)
c) Weakly curved background with disturbing neighbouring lines: approach $H(\vartheta) = a_1 + a_2\vartheta + a_3\vartheta^2 + a_4\vartheta^3$ (cubic parabola, $m = 4$)
d) Strongly curved background, which can be present with strongly overlapping lines, for instance, with alpha radiation: approach according to equation (C.6)

In cases a, b, and c, the scaling variable ϑ is required to be linearly assigned to the channel number.

In cases a and b, it is suitable for the background determination to introduce three adjacent channel regions A_1, B, and A_2 in the following way.

Region B comprises all the channels belonging to the line and has the total content n_g and the width t_g. If the line shape can be assumed as a Gaussian curve with the full width h at half maximum, then region B has to be placed as symmetrically as possible over the line. The following should be chosen:

$$t_\mathrm{g} \approx 2{,}5\,h \qquad \text{(C.9)}$$

if fluctuations of the channel assignment cannot be excluded or the background does not dominate, for instance, with pronounced lines. In case of a dominant background, the most favourable width

$$t_\mathrm{g} \approx 1{,}2\,h \qquad \text{(C.10)}$$

has to be specified for region B. This region then covers approximately the portion $f = 0{,}84$ of the line area (see also C.4). In general, $f = 2\Phi(v\sqrt{2\ln 2}) - 1$, if $t_\mathrm{g} = v\,h$ with a chosen factor v.

In principle, the full width h at half maximum has to be determined from the resolution of the measuring system or under the same measurement conditions by means of a reference sample emitting the line to be investigated strongly enough, or from neighbouring lines with comparable shapes and widths. Region B must comprise an integer number of channels, so that t_g has to be rounded up accordingly.

Regions A_1 and A_2, bordering region B below and above, have to be specified with the same widths $t = t_1 = t_2$ in case b only. The total width $t_0 = t_1 + t_2 = 2t$ has to be chosen as large as possible, but at most so large that the background shape over all regions can still be taken as approximately constant (case a) or linear (case b). n_1 and n_2 are the total contents of all channels of regions A_1 and A_2, respectively. Moreover, $n_0 = n_1 + n_2$.

Hence follows for cases a and b:

$$z_0 = c_0 n_0 \;\; ; \quad u^2(z_0) = c_0^2 n_0 \;\; ; \quad c_0 = t_\mathrm{g}/t_0 \;\; . \qquad \text{(C.11)}$$

$\widetilde{u}^2(\eta)$ follows from equation (C.5).

Instead, in case c, five adjacent channel regions A_1, A_2, B, A_3, and A_4 have to be introduced in the way described above with the same widths t of the regions A_i (see Figure C.1). With the sum $n_0 = n_1 + n_2 + n_3 + n_4$, i.e. the total content of all channels of regions A_i, with their total width $t_0 = 4t$, and with the auxiliary quantity $n_0' = n_1 - n_2 - n_3 + n_4$, the following is then valid:

$$\begin{aligned} &z_0 = c_0 n_0 - c_1 n_0' \;\; ; \quad u^2(z_0) = (c_0^2 + c_1^2) n_0 - 2 c_0 c_1 n_0' \;\; ; \\ &c_0 = t_\mathrm{g}/t_0 \;\; ; \quad c_1 = c_0 \cdot (4/3 + 4c_0 + 8c_0^2/3)/(1 + 2c_0) \end{aligned} \qquad \text{(C.12)}$$

and $\widetilde{u}^2(\eta)$ follows from equation (C.5). Two numerical examples of case c are treated in D.5.

In case d, m adjacent regions A_i have to be introduced in the same way, with approximately half of them arranged below and above region B. The regions A_i need not have the same widths. The power functions ϑ^{k-1} have to be chosen to some extent as above as the functions $\psi_k(\vartheta)$. For the same purpose, the functional shapes of the disturbing neighbouring lines that have to be considered should also be chosen as far as possible and known. Then, one has to proceed according to C.1 and $\widetilde{u}^2(\eta)$ again follows from equation (C.5).

After $\widetilde{u}^2(\eta)$ has been calculated in all cases according to equation (C.5), the characteristic limits result with equation (C.4) and according to Section 6.

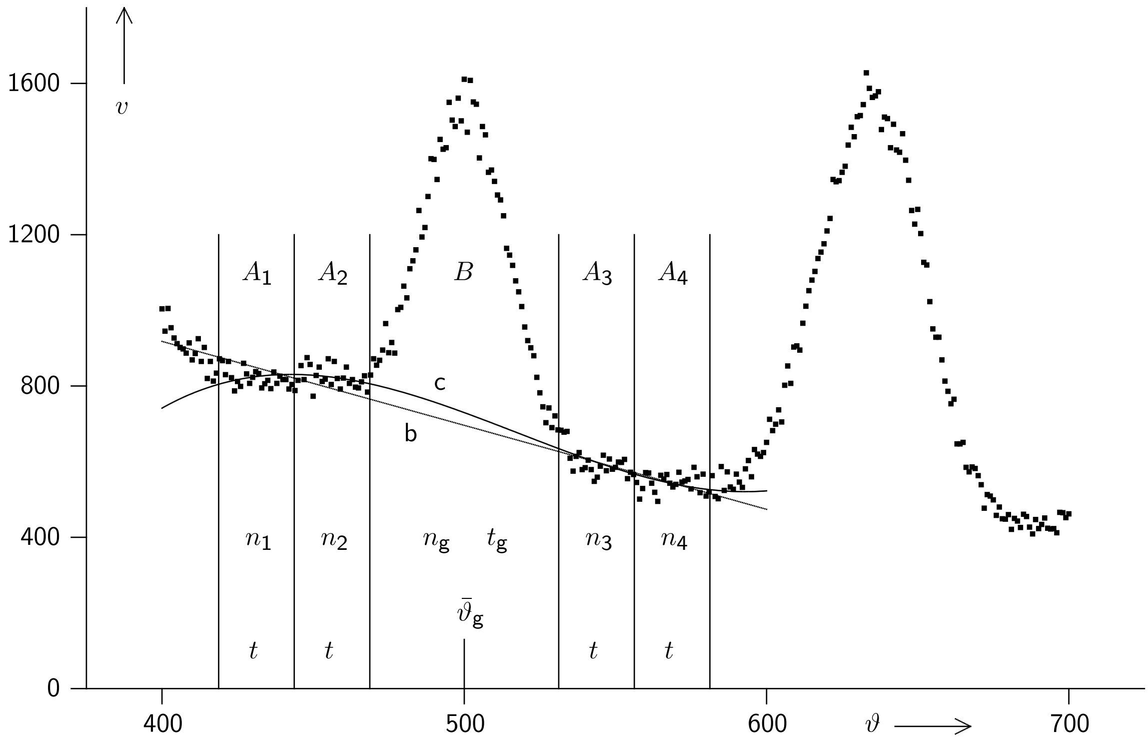

Figure C.1: Arrangement of the channel regions for the determination of the background of a line

Arrangement scheme of the adjacent channel regions A_i $(i = 1,2,3,4)$ in the multi-channel spectrum for the determination of a weakly curved background of a line in region B (case c). The regions A_i have the contents n_i and the same width t, region B has the content n_g and the width $t_g = 2{,}5\,h$ with the full width h at half maximum. The abscissa ϑ, for instance, energy or time, is assigned to the channel number and $\bar{\vartheta}_g$ is its value in the middle of region B. The ordinate v denotes the counted content of each of the channels. With a constant or linear background, only two regions A'_i arranged in the order A'_1, B, A'_2 are needed (cases a and b). The straight line b and the cubic parabola c represent the background shape of the line in the spectrum. They are determined according to C.3 for cases b and c, respectively. For case b, regions A_1 and A_2 have been combined to form A'_1 with the width $2t$ and, likewise, regions A_3 and A_4 to form A'_2. The straight line b does not fulfill the chi-square condition (see D.5).

C.3 Obtaining the regions for determining the background

The regions A_i for background determination can be obtained by performing a test on whether or not the function $H(\vartheta)$ can represent the background shape. For this purpose and with the total number $M > m$ of all channels of regions A_i, with the counted content v_j of channel j $(j = 1, \ldots, M)$ of these regions, with the value $\bar{\vartheta}_j$ of the scaling variable ϑ assigned to the middle of the channel j, and with the channel width $\Delta\vartheta_j$, the test quantity

$$\chi^2 = \sum_{j=1}^{M} \frac{(H(\bar{\vartheta}_j\,; a_1, \ldots, a_m)\,\Delta\vartheta_j - v_j)^2}{v_j + 1} \tag{C.13}$$

is calculated. Then it is ascertained whether or not

$$|\chi^2 - M + m| \leq k_{1-\delta/2}\sqrt{2(M-m)}\ . \tag{C.14}$$

The error probability $\delta = 0{,}05$ is recommended. Depending on whether the chi-square condition according to equation (C.14) for the compatibility of the function $H(\vartheta)$ with the measured background shape in the regions A_i of the spectrum is fulfilled or not, the regions A_i and, thus, M have to be enlarged or reduced, respectively, and the test has to be repeated until maximum regions still compatible with the condition are found.

If functional values $H(\vartheta)$ are negative in the regions A_i and B, then the procedure is not applicable in the way described here. For the denominator $v_j + 1$ in equation (C.13), see under equation (G.1).

In cases a to c, the function $H(\vartheta)$ can be explicitly specified:

$$\text{case a)}\quad H(\vartheta) = \frac{n_0}{t_0} \;; \tag{C.15}$$

$$\text{case b)}\quad H(\vartheta) = \frac{n_0}{t_0} + \frac{4(n_2 - n_1)(\vartheta - \bar{\vartheta}_\mathrm{g})}{t_0(2t_\mathrm{g} + t_0)} \;; \tag{C.16}$$

$$\text{case c)}\quad H(\vartheta) = a_1 + a_2(\vartheta - \bar{\vartheta}_\mathrm{g}) + a_3(\vartheta - \bar{\vartheta}_\mathrm{g})^2 + a_4(\vartheta - \bar{\vartheta}_\mathrm{g})^3 \tag{C.17}$$

where $\bar{\vartheta}_\mathrm{g}$ is the value of ϑ assigned to the middle of region B and, moreover,

$$\begin{aligned} a_1 &= \frac{n_0}{t_0} - \frac{4\,n_0'\,(t_\mathrm{g}^2 + t_\mathrm{g}t_0 + t_0^2/3)}{t_0^2(2t_\mathrm{g} + t_0)} \;; \quad a_2 = 16\,\frac{n_3 - n_2}{t_0(4t_\mathrm{g} + t_0)} - \frac{a_4}{32}((2t_\mathrm{g} + t_0)^2 + (2t_\mathrm{g})^2) \;; \\ a_3 &= \frac{16\,n_0'}{t_0^2(2t_\mathrm{g} + t_0)} \;; \quad a_4 = 256\,\frac{(n_4 - n_1)(4t_\mathrm{g} + t_0) - (n_3 - n_2)(4t_\mathrm{g} + 3t_0)}{t_0^2(4t_\mathrm{g} + t_0)(4t_\mathrm{g} + 2t_0)(4t_\mathrm{g} + 3t_0)} \;. \end{aligned} \tag{C.18}$$

As a numerical example, Figure C.1 shows a section of a multi-channel spectrum, recorded using a NaI detector, with the background shapes calculated according to cases b and c. See D.5.2 for more details.

C.4 Extending applications

From the net line intensity obtained according to C.1 and C.2 and in combination or comparison with further quantities (for instance, calibration, correction or influence quantities or conversion factors such as sample mass, emission or response probability), another measurand of interest has often to be calculated. This can be, for instance, an activity (concentration) or the quotient of the net line intensity and the net intensity of a reference line in the same spectrum or the net intensity of the same line in a reference spectrum. In such cases, after the calculations according to C.1 and C.2 have been carried out, one has to proceed in essence according to 5.2 and 5.3 as follows.

In 5.2 and 5.3, the measurand Y of interest and the input quantities X_i appear. They have to be specified according to the following equations, where on the left-hand side one of the aforementioned quantities and on the right-hand side the respective quantity according to C.1 are found.

If Y is an activity (concentration) or an analogous quantity, then $X_1 = X_\mathrm{g}$ and $X_2 = Z_0$ and $X_3 = 1$ are set. Moreover, $x_5 = 1$ or 0,84 and $u(x_5) = 0$, if equations (C.9) and (C.10), respectively, are used. Further input quantities X_i are specified as conversion factors.

If $Y = Y_1/Y_2$ is the quotient of the net line intensity Y_1, determined according to C.1 and C.2, and the likewise determined net intensity Y_2 of a reference line in the same or a different spectrum, then $X_1 = Y_1$ and $X_2 = 0$ and $X_5 = Y_2$ are specified.

For correcting a spectrometric superposition of the line of interest by a disturbing line L with the same energy, but from a different nuclide, one has to proceed in a way similar to the preceding paragraph. Then $X_1 = Y_1$ is the net intensity sum of both lines, and $X_2 = Y_2$ is the net intensity of a line of the disturbing nuclide that serves as a reference. With the presumption that the spectrum of this nuclide can be separately measured free from the line of interest, for instance, on a blank, two cases must be differentiated. In the first case, the disturbing line L itself serves as a reference. Then $x_3 = t_1/t_2$ and $u(x_3) = 0$ for X_3 have to be specified, where t_1 and t_2 are the measurement durations of the spectra. In the second case, another line L' of the disturbing nuclide in the spectrum to be examined serves as a reference. Then the net intensities i and i' of the lines L and L', respectively, and the associated standard uncertainties $u(i)$ and $u(i')$ have to be determined from the separately measured spectrum, and the following has to be specified:

$$x_3 = \frac{i}{i'} \;; \quad u^2(x_3) = x_3^2 \cdot \Big(\frac{u^2(i)}{i^2} + \frac{u^2(i')}{i'^2}\Big) \;. \tag{C.19}$$

Annex D
(informative)
Application examples

D.1 General aspects

This Annex D contains numerical examples of the applications treated in the normative Annexes B and C. The respective equations used for the calculations are referred to. In all examples, y, $u(y)$ and $\widetilde{u}(\eta)$ are first determined and then the characteristic limits as well as the best estimate of the measurand with the associated standard uncertainty are calculated according to the equations given in Section 6 or A.5 to A.8 and by applying Annex E.

The data in Tables D.1 to D.4 are often given with more digits than meaningful, so that the calculations can also be reconsidered and verified with higher accuracy, in particular, for testing computer programs under development. Some intermediate values, which must be calculated in a more complicated way, are also given for test purposes.

D.2 Example 1: Measurement of the surface activity concentration by means of the wipe test

D.2.1 Counting measurement

For the examination of a surface contamination by means of the wipe test, the measurand Y is the surface activity concentration A_F (activity divided by the wiped area, see ISO 31-0). For this task, the characteristic limits, the best estimate and the associated standard uncertainty are to be calculated. The model of the evaluation in this case reads according to equation (4)

$$Y = A_F = \frac{X_1 - X_2}{X_5\,X_7\,X_9} = \frac{\varrho_\mathrm{g} - \varrho_0}{F\,\kappa\,\varepsilon} \quad . \tag{D.1}$$

$X_1 = \varrho_\mathrm{g}$ is the gross count rate and $X_2 = \varrho_0$ is the background count rate, $X_5 = F$ is the wiped area, $X_7 = \kappa$ is the detection efficiency, and $X_9 = \varepsilon$ is the wiping efficiency, i.e. the fraction of the wipeable activity for the material of the surface to be examined.

After the counting measurements of the gross effect and of the background effect are carried out with the respective measurement durations t_g and t_0, the respective numbers n_g and n_0 of the recorded events are available. These numbers are used according to 5.2.2 to specify the estimate $x_1 = r_\mathrm{g} = n_\mathrm{g}/t_\mathrm{g}$ with $u^2(x_1) = n_\mathrm{g}/t_\mathrm{g}^2 = r_\mathrm{g}/t_\mathrm{g}$ for the the gross count rate $X_1 = \varrho_\mathrm{g}$ and $x_2 = r_0 = n_0/t_0$ with $u^2(x_2) = n_0/t_0^2 = r_0/t_0$ for the background count rate $X_2 = \varrho_0$. These specifications apply to measurements with time preselection.

The detection efficiency $\kappa =$ 0,31 is determined using a calibration source with a certified relative standard uncertainty of 5 %. On the assumption that the statistical contribution to the measurement uncertainty of the detection efficiency is negligible, $u(\kappa) =$ 0,0155 results.

The wiping efficiency ε of the wipe test is known from previous measurements to be randomly distributed between 0,06 and 0,62. This yields the mean estimate $\varepsilon =$ 0,34 and the associated standard uncertainty $u(\varepsilon) = \Delta\varepsilon/\sqrt{12} =$ 0,16 by specifying a rectangular distribution over the region of the possible values of ε with the width $\Delta\varepsilon =$ 0,56 (see 5.2.2, last but one paragraph).

The relative standard uncertainty of the wiped area $F =$ 100 cm^2 is given as 10 % from experience, leading to $u(F) =$ 10 cm^2.

For the input data, specifications, some intermediate values, and results, see Table D.1. The results are calculated according to 5.2.2, 5.3.2 and Section 6. In particular, equations (6), (9), and (14) are used for y, $u(y)$ and $\widetilde{u}(\eta)$, respectively, where $x_3 = 1$ and $u(x_3) = 0$ are set because X_3 is not involved in the model. Some standard uncertainties are not given in Table D.1 since they are not explicitly needed for the equations.

The decision threshold and the detection limit, obtained according to equation (16) if the counting measurements are regarded as carried out with preselection of counts, are given in brackets in the last but one column of Table D.1. All the other results do not depend thereon.

D.2.2 Measurement using a ratemeter

The measurement of the count rate can also be carried out using a ratemeter (see B.3). In contrast to D.2.1, $u^2(x_1) = r_\mathrm{g}/(2\tau_\mathrm{g})$ and $u^2(x_2) = r_0/(2\tau_0)$ here apply. In Table D.1, the input data of the ratemeter measurement are fictitiously chosen such that the primary measurement result y is almost unchanged when compared with that of the counting measurement. The relaxation time constants strongly influence the decision threshold and the detection limit. Their values $\tau_g = \tau_0 = 15$ s are chosen too small and therefore make the measurement procedure unsuitable for the measurement purpose since $y^* > y_r$. The choice $\tau_g = \tau_0 = 20$ s would here already afford relief.

Table D.1: Input data, intermediate values and results of example 1

Input data and specifications quantity	symbol	value	standard uncertainty
counting measurement, gross effect:			
number of recorded events	n_g	2591	
measurement duration	t_g	360 s	neglected
counting measurement, background effect:			
number of recorded events	n_0	41782	
measurement duration	t_0	7200 s	neglected
ratemeter measurement, gross effect:			
count rate	r_g	7,20 s^{-1}	
relaxation time constant	τ_g	15 s	not needed
ratemeter measurement, background effect:			
count rate	r_0	5,80 s^{-1}	
relaxation time constant	τ_0	15 s	not needed
wiped area	F with $u(F)$	100 cm^2	10 cm^2
detection efficiency	κ with $u(\kappa)$	0,31	0,0155
wiping efficiency	ε with $u(\varepsilon)$	0,34	0,16
probabilities	α, β, γ	0,05	–
guideline value	η_r	0,5 Bq cm^{-2}	–

Intermediate values quantity and calculation	value [1]	value [2]
$w = 1/(F\kappa\varepsilon)$ according to equation (7)	0,0949 cm^{-2}	
$u^2_{rel}(w) = u^2(F)/F^2 + u^2(\kappa)/\kappa^2 + u^2(\varepsilon)/\varepsilon^2$ according to equation (10)	0,2340	
$\omega = \Phi(y/u(y))$ according to equation (E.1)	0,9784	0,9309
$p = \omega \cdot (1 - \gamma/2)$	0,9539	0,9076
$q = 1 - \omega\gamma/2$	0,9755	0,9767
k_p according to equation (E.2)	1,6843	1,3262
k_q according to equation (E.2)	1,9623	1,9904

Results measurand : quantity	Y symbol	A_F [1] value in Bq cm^{-2}	A_F [2]
primary measurement result	y	0,1323	0,1328
standard uncertainty associated with y	$u(y)$	0,0654	0,0896
decision threshold	y^*	0,0203 (0,0183)	0,0970
measurement effect present ?	$y > y^*$?	yes	yes
detection limit	η^*	0,1126 (0,1033)	0,5521
measurement procedure suitable ?	$\eta^* \leq \eta_r$?	yes	no
lower confidence limit	η_l	0,0221	0,0140
upper confidence limit	η_u	0,2611	0,3112
best estimate of the measurand	z	0,1357	0,1456
standard uncertainty associated with z	$u(z)$	0,0617	0,0785

[1]) Counting measurement with time preselection. In brackets: changed values from an equivalent counting measurement with preselection of counts
[2]) Ratemeter measurement

D.3 Example 2: Measurement of the specific activity of ^{90}Sr after chemical separation

D.3.1 Unknown influence of sample treatment

A soil contamination with ^{90}Sr can be examined by chemical separation of this nuclide and subsequent measurement of the radiation from the beta decay of ^{90}Sr via ^{90}Y to ^{90}Zr. (A possible influence on the measurement by ^{89}Sr is here neglected.) The measurand Y is the specific activity A_M (activity divided by the total mass of the sample, see ISO 31-9) for which the characteristic limits, the best estimate, and the associated standard uncertainty are to be calculated. The measurement is randomly influenced by sample treatment because of the chemical separation. Therefore, one has to proceed according to B.4. For determining and reducing the influence, several soil samples of the same kind, blanks and, if necessary, also reference samples are separately tested. The results for the respective samples are then averaged and analysed regarding the measurement uncertainty.

The model of the evaluation reads in this case according to equation (4)

$$Y = A_M = \frac{X_1 - X_2}{X_5\, X_7\, X_9} = \frac{\overline{\varrho}_\mathrm{g} - \overline{\varrho}_0}{M\, \kappa\, \varepsilon} \ . \qquad \text{(D.2)}$$

$X_1 = \overline{\varrho}_\mathrm{g}$ is the mean gross count rate of the samples and $X_2 = \overline{\varrho}_0$ is the mean background count rate of the blanks, $X_5 = M$ is the sample mass set to be identical for all samples, blanks, and reference samples, $X_7 = \kappa$ is the detection efficiency of the detector used for the counting measurement of the beta radiation in the current measurement geometry, and $X_9 = \varepsilon$ is the chemical yield of ^{90}Sr separation. There is no formal difference between equation (D.2) and equation (D.1), but they must be distinguished because of the different interpretations of the quantities X_i and, in essence, due to the count rates being averaged or not.

After the counting measurements of the gross effect on m_g samples to be tested and of the background effect on m_0 blanks are carried out with the preselected measurement durations t_g and t_0, respectively, the numbers $\overline{n}_\mathrm{g}$ and $\overline{n}_0$ of the recorded events averaged according to equation (B.7) are available. This first yields the estimates $x_1 = \overline{n}_\mathrm{g}/t_\mathrm{g}$ and $x_2 = \overline{n}_0/t_0$ of the respective mean count rates $X_1 = \overline{\varrho}_\mathrm{g}$ and $X_2 = \overline{\varrho}_0$ according to equation (B.8). Moreover, the empirical variances s_g^2 and s_0^2 of the counting results have to be formed according to equation (B.7). These yield according to equation (B.9) the squares of the standard uncertainties $u^2(x_1) = s_\mathrm{g}^2/(m_\mathrm{g} t_\mathrm{g}^2)$ and $u^2(x_2) = s_0^2/(m_0 t_0^2)$ associated with the estimates of the count rates. With these results, the estimate y of the measurand $Y = A_M$ and the associated standard uncertainty $u(y)$ then have to be calculated according to 5.2.2 and, in particular, according to equations (6) and (9), respectively. $x_3 = 1$ and $u(x_3) = 0$ must be set since X_3 is not involved in the model. Finally, the confidence limits, the best estimate z and the associated standard uncertainty $u(z)$ can be calculated according to 6.4 and 6.5, in this example as approximations according to equations (29) and (32) because of $y \geq 4u(y)$.

The next step concerns the function $\widetilde{u}^2(\eta)$. The standard uncertainty $u(x_1)$ is not available as a function $h_1(x_1)$. But the interpolation according to equation (19) can instead be used. However, $\widetilde{u}^2(0)$ is needed for this and obtained as follows: setting $y = \eta = 0$ in equation (9) first yields $\widetilde{u}^2(0) = w^2 \cdot (u^2(x_1) + u^2(x_2))$. Moreover, for $\eta = 0$ according to 5.3.2, the variance s_g^2 has to be replaced by s_0^2. This leads with equation (B.9) to $u^2(x_1) = s_0^2/(m_\mathrm{g} t_\mathrm{g}^2)$ and finally to

$$\widetilde{u}^2(0) = w^2 s_0^2 \cdot (1/(m_\mathrm{g} t_\mathrm{g}^2) + 1/(m_0 t_0^2)) \ . \qquad \text{(D.3)}$$

The decision threshold then follows from equation (21) and the detection limit with the interpolation according to equation (19) from equations (22) or (25).

For the input data, specifications, some intermediate values, and results, see Table D.2. (The values bracketed there as well as the results in the last column belong to D.3.2.) The guideline value is taken from a directive on monitoring environmental radioactivity.

D.3.2 Known influence of sample treatment

The random influence of sample treatment is sometimes already known from previous measurements, namely from measurements on reference samples or on other samples. The latter should be similar to the current samples and be measured under similar conditions so that they can be taken as reference samples although they need not be examined specifically for reference purposes.

One can also proceed in this case according to the equations in B.4.3. For the data of the calculation example, see also Table D.2. To enable a comparison, the same input data as in D.3.1 are used here and, moreover, the counting results of the reference samples are given in brackets. In contrast to D.3.1, the variance $u^2(x_1)$ according to equation (B.15) is known as a function $h_1^2(x_1)$ of x_1. For obtaining $\widetilde{u}^2(\eta)$, the estimate y in equation (B.16) is first replaced by η and then $u^2(x_1)$ and $u^2(x_2)$ by the expressions according to equations (B.15) and (B.14), respectively. This leads with $x_1 = \eta/w + x_2$ and ϑ^2 according to equation (B.13) to

$$\widetilde{u}^2(\eta) = w^2 \cdot \left((x_1/t_\mathrm{g} + \vartheta^2 x_1^2)/m_\mathrm{g} + (x_2/t_0 + \vartheta^2 x_2^2)/m_0 \right) + \eta^2 u_\mathrm{rel}^2(w) \ . \qquad \text{(D.4)}$$

The results for D.3.1 and D.3.2 shown in Table D.2 agree in essence, as must be expected. For the influence parameter ϑ, the value 0,1377 $<$ 0,2 acceptable according to B.4.3 results. The decision threshold and the detection limit are in the case of D.3.2 slightly smaller than those of D.3.1. This may be due to the additional information from the reference samples.

Table D.2: Input data, intermediate values and results of example 2

Input data and specifications quantity	symbol	value (in brackets for D.3.2)	standard uncertainty
number of samples, blanks and reference samples	m_{g}, m_0, m_{r}	5, 5, (20)	
numbers of recorded events:			
samples (gross effect)	$n_{\mathsf{g},i}$	1832, 2259, 2138, 2320, 1649	
blanks (background effect)	$n_{0,i}$	966, 676, 911, 856, 676	
reference samples	$n_{\mathsf{r},i}$	(74349, 67939, 88449, 83321, 66657, 64094, 74348, 93576, 56402, 66785, 78194, 69221, 63965, 70503, 74220, 97422, 74476, 71784, 68235, 74989)	
measurement durations (general)	t_{g}, t_0, t_{r}	30000 s	neglected
sample mass (general)	M with $u(M)$	0,100 kg	0,001 kg
detection efficiency	κ with $u(\kappa)$	0,51	0,02
chemical yield of ^{90}Sr separation	ε with $u(\varepsilon)$	0,57	0,04
probabilities	α, β, γ	0,05	–
guideline value	η_{r}	0,5 Bq kg^{-1}	–

Intermediate values quantity and calculation	symbol	value (in brackets for D.3.2)
mean values	$\overline{n}_{\mathsf{g}}$, $\overline{n}_0$, $\overline{n}_{\mathsf{r}}$	2039,6; 817,00; (73946,5)
and empirical standard deviations according to equation (B.7)	s_{g}, s_0, s_{r}	288,14; 134,46; (10185,0)
influence parameter according to equation (B.13)	$\vartheta = ((s_{\mathsf{r}}^2 - \overline{n}_{\mathsf{r}})/\overline{n}_{\mathsf{r}}^2)^{1/2}$	(0,1377)

Results measurand : quantity	Y symbol	A_M (D.3.1) value in Bq kg^{-1}	A_M (D.3.2) value in Bq kg^{-1}
primary measurement result	y	1,4019	1,4019
standard uncertainty associated with y	$u(y)$	0,1987	0,1942
decision threshold	y^*	0,1604	0,1384
measurement effect present ?	$y > y^*$?	yes	yes
detection limit	η^*	0,3786	0,3053
measurement procedure suitable ?	$\eta^* \leq \eta_{\mathsf{r}}$?	yes	yes
lower confidence limit	η_{l}	1,0124	1,0213
upper confidence limit	η_{u}	1,7914	1,7825
best estimate of the measurand	z	1,4019	1,4019
standard uncertainty associated with z	$u(z)$	0,1987	0,1942

D.4 Example 3: Measurement of the activity concentration and of its increase during accumulation on a filter

A radiochemical laboratory is working exclusively with ^{131}I. Due to legal requirements, the activity concentration of the exhaust air must not exceed the value of 20 Bq m^{-3}. For monitoring compliance with this condition, part of the exhaust air is passed through a filter. The activity of the filter is continuously measured at measurement intervals of duration t with a counting measuring instrument. This implies a case according to B.5. The measurand Y of interest is, on the one hand, the activity concentration $A_{V,j}$ of the exhaust air during the measurement interval j (see B.5.2) and, on the other hand, also the increase $\Delta A_{V,j}$ of the activity concentration $A_{V,j}$ in comparison to the mean activity concentration $\overline{A}_{V,j}$ of m preceding measurement intervals (see B.5.3). For each of these cases, the respective characteristic limits, the best estimate, and the associated standard uncertainty are to be calculated.

The model for the activity concentration $A_{V,j}$ is given in equation (B.18), the model for the increase $\Delta A_{V,j}$ of the activity concentration in equation (B.26). They do not differ formally, but merely in the interpretations and approaches of X_2 according to B.5.2 and B.5.3, respectively.

For the input data, specifications, some intermediate values, and results, see Table D.3. The numbers n_j from 26 measurement intervals from $j = 0$ to 25 are available. The measurement interval $j = 25$ is to be examined. Therefore, $m = 24$ is set, and only the numbers n_j for $j = 25$, 24 and 0 are needed, but not explicitly the associated standard uncertainties $u(n_j) = \sqrt{n_j}$. For the approaches of the values x_1 and $u^2(x_1)$ for X_1 as well as x_2 and $u^2(x_2)$ for X_2, see B.5. The guideline value $\eta_r = 2$ Bq m^{-3} is specified for $A_{V,j}$, so that activity concentrations of at least 10 % of the value required by law can still be recognized. For $\Delta A_{V,j}$, the guideline value $\eta_r = 0{,}2$ Bq m^{-3} is chosen, so that technical measures can be initiated in time for reducing the activity concentration below 10 % of the value required by law. The results are calculated by means of the mentioned models according to Annex A and B.5, especially by application of equations (B.18) to (B.28). For $Y = A_{V,25}$ in B.5.2, the approximations according to equations (29) and (32) are used because of $y \geq 4u(y)$.

Table D.3: Input data, intermediate values and results of example 3

Input data and specifications quantity	symbol	value	standard uncertainty
number of recorded events in the measurement intervals 25, 24 and 0 ($j = 25$)	$n_j = n_{25}$	15438	
	$n_{j-1} = n_{24}$	14356	
	n_0	2124	
duration of a measurement interval	t	3600 s	neglected
volume	V with $u(V)$	3,00 m^3	0,01 m^3
calibration factor	ε with $u(\varepsilon)$	0,37	0,02
probabilities	α, β, γ	0,05	–
guideline values for $A_{V,j}$ and $\Delta A_{V,j}$	η_r	2,0 and 0,2 Bq m^{-3}	–

Intermediate values quantity and calculation		value	standard uncertainty
x_2 with $u(x_2)$ according to equation	(B.20) for B.5.2	3,9878 Bq	0,0333 Bq
	(B.28) for B.5.3	4,1294 Bq	0,0347 Bq

Results measurand :	Y	$A_{V,25}$ (B.5.2)	$\Delta A_{V,25}$ (B.5.3)
quantity	symbol	value in Bq m^{-3}	
primary measurement result	y	0,2708	0,1432
standard uncertainty associated with y	$u(y)$	0,0456	0,0448
decision threshold	y^*	0,0697	0,0718
measurement effect present ?	$y > y^*$?	yes	yes
detection limit	η^*	0,1413	0,1455
measurement procedure suitable ?	$\eta^* \leq \eta_r$?	yes	yes
lower confidence limit	η_l	0,1814	0,0560
upper confidence limit	η_u	0,3602	0,2310
best estimate of the measurand	z	0,2708	0,1433
standard uncertainty associated with z	$u(z)$	0,0456	0,0446

D.5 Examples 4 and 5: Measurement of the specific activity via the intensity of a line on a weakly curved background in a gamma spectrum

D.5.1 Example 4: Measurement using a germanium detector

In the gamma spectrum of a soil sample recorded by means of a Ge detector, there is a line assigned to the nuclide to be examined and located at channel 927 on a dominant, weakly curved background. The measurand Y is the specific activity A_M of the sample (activity divided by the total mass of the sample, see ISO 31-9) and has to be calculated from the net intensity (net area) of the line. For this measurand, the characteristic limits, the best estimate, and the associated standard uncertainty have to be determined.

Case c of C.2 is present. As known from energy calibration, the energetic width of a channel amounts to 0,4995 keV, and the energetic full width at half maximum of the line is 2,0 keV. This corresponds to a full width at half maximum of $h = 4{,}00$ channels. According to equation (C.10), $t_\mathrm{g} \approx 1{,}2\,h = 4{,}8$ is set as the width of region B. The region of channels 925 to 929 with the width $t_\mathrm{g} = 5$ and located symmetrically to channel 927 is therefore specified as region B (see Figure C.1). This region thus covers in this case approximately the portion $f = 86$ % of the line area (see also under equation (C.10)).

For each of the four regions A_i bordering region B on both sides for the determination of the weakly curved background, the width $t = 13$ is chosen according to C.3. The total width thus amounts to $t_0 = 52$. This width cannot be enlarged since there is another possible line at channel 958 with the same full width at half maximum and therefore located in channels 956 to 960. Thus, at most the 26 channels 930 to 955 remain for the regions A_3 and A_4.

For the input data, specifications, some intermediate values, and results, see Table D.4. The results are calculated on the basis of the following model according to Annex A and C.2. Especially, equations (C.3), (C.4), (C.5), (C.10), and (C.12) are used. The model reads

$$Y = A_M = \frac{X_1 - X_2}{X_5\,X_7\,X_9\,X_{11}\,X_{13}} = \frac{X_\mathrm{g} - Z_0}{T\,f\,M\,\varepsilon\,i} \ . \qquad \text{(D.5)}$$

$X_1 = X_\mathrm{g}$ is the estimator of the gross effect in region B, $X_2 = Z_0$ is the estimator of the background effect, i.e. of the background contribution to the line in region B, and $X_5 = T$ is the measurement duration. The correction factor $X_7 = f$ takes into account that region B does not completely cover the line in case of a dominant background. For f, see above and also under equation (C.10). The standard uncertainty of f is neglected because f, if necessary, can be calculated exactly to an arbitrary number of digits. Moreover, $X_9 = M$ is the sample mass, $X_{11} = \varepsilon$ is the detection efficiency of the detector measured with $f = 1$, and $X_{13} = i$ is the photon emission probability of the gamma line. The values of M and ε and the associated standard uncertainties $u(M)$ and $u(\varepsilon)$ were determined in previous investigations. The value of i and the associated standard uncertainty $u(i)$ are taken from a tabular compilation of decay data of radioactive nuclides. The guideline value η_r is specified by a directive on monitoring of environmental radioactivity.

For X_1, the values $x_1 = n_\mathrm{g}$ and $u^2(x_1) = n_\mathrm{g}$ are set (see C.1 and G.1). It should be noted here that $X_1 = X_\mathrm{g}$ does not estimate a count rate, but instead the parameter of a Poisson distribution. Therefore, the measurement duration T appears in the denominator of equation (D.5). For the values z_0 and $u^2(z_0)$ for $X_2 = Z_0$, see equation (C.12).

D.5.2 Example 5: Measurement using a sodium iodide detector

Figure C.1 shows a section of a gamma spectrum recorded using a NaI detector. There is a line of interest located with its center $\overline{\vartheta}_\mathrm{g}$ at channel 500 on a non-dominant, weakly curved background. The measurand Y is the net intensity I (net area) of the line. For this measurand, the characteristic limits, the best estimate, and the associated standard uncertainty have also to be determined.

Again, case c of C.2 is present. The full width at half maximum of the line approximately amounts to $h = 25$ channels. Thus, $t_\mathrm{g} \approx 2{,}5\,h = 62{,}5$ has to be set as the width of region B according to equation (C.9). Therefore, the region of channels 469 to 531 with the width $t_\mathrm{g} = 63$ and located symmetrically to channel 500 is specified as region B (see Figure C.1). This region thus covers in this case almost $f = 100$ % of the line area.

For each of the four regions A_i bordering region B on both sides for the determination of the weakly curved background, the width $t = 25$ is chosen according to C.3. The total width thus amounts to $t_0 = 100$. This width cannot be enlarged because of the increasing background above channel 581 due to the second line shown in Figure C.1 and below channel 419.

For the input data, specifications, some intermediate values, and results, see Table D.4. The results are calculated on the basis of the following model as in example 4. The model here has a simpler form and reads

$$Y = I = X_1 - X_2 = X_\mathrm{g} - Z_0 \qquad \text{(D.6)}$$

so that $w = 1$ and $u_{\mathrm{rel}}(w) = 0$. For the input quantities $X_1 = X_{\mathrm{g}}$ and $X_2 = Z_0$, see also example 4. A guideline value is not specified. Because of $y \geq 4u(y)$ in the present case, the approximations according to equations (29) and (32) are used.

As shown in Figure C.1, both a straight line with $m = 2$ (case b) and a cubic parabola with $m = 4$ (case c) are adjusted to the spectrum background in the regions A_i according to C.3. In the case of the straight line, the regions A_1 and A_2 are combined, the regions A_3 and A_4 as well. With $M = t_0 = 100$ and $\delta = 0{,}05$ and according to equation (C.14), the standardized chi-square $\chi_{\mathrm{s}}^2 = |\chi^2 - M + m| / \sqrt{2(M - m)} = 4{,}18 > k_{1-\delta/2} = 1{,}96$ follows for the straight line, but the value $1{,}21 < 1{,}96$ for the parabola. The straight line therefore cannot be accepted because the chi-square condition is not fulfilled. The numbers of the events recorded in the individual spectrum channels are not attached for reasons of space.

Table D.4: Input data, intermediate values and results of examples 4 and 5

Input data and specifications of example 4			
quantity	symbol	value	channels
energetic channel width		0,4995 keV	
energetic full width at half maximum of the line		2,0 keV	
number of recorded events in			
region A_1	n_1	3470	899 to 911
region A_2	n_2	3373	912 to 924
region B	n_{g}	1440	925 to 929
region A_3	n_3	3343	930 to 942
region A_4	n_4	3208	943 to 955
width of a region A_i	t	13	
width of region B	t_{g}	5	
			standard uncertainty
measurement duration	T	21600 s	neglected
correction factor	f	0,8585	neglected
sample mass	M with $u(M)$	1,000 kg	0,001 kg
detection efficiency	ε with $u(\varepsilon)$	0,060	0,004
photon emission probability	i with $u(i)$	0,98	0,02
probabilities	α, β, γ	0,05	–
guideline value	η_{r}	0,5 Bq kg^{-1}	–

Input data and specifications of example 5			
quantity	symbol	value	channels, comments
full width at half maximum of the line	h	25	
number of recorded events in			
region A_1 } A_1 for straight line	n_1	20556	419 to 443
region A_2 } A_1 for straight line	n_2	20549	444 to 468
region B	n_{g}	72691	469 to 531
region A_3 } A_2 for straight line	n_3	14965	532 to 556
region A_4 } A_2 for straight line	n_4	13580	557 to 581
width of a region A_i	t	25	
width of region B	t_{g}	63	
probabilities	α, β, γ, δ	0,05	
guideline value	η_{r}	–	not specified

(continued)

Table D.4 (completed)

Intermediate values quantity and calculation		**Example 4** value	**Example 5** value
$n_0 = n_1 + n_2 + n_3 + n_4$		13394	69650
$n_0' = n_1 - n_2 - n_3 + n_4$		–38	–1378
total width $t_0 = 4t$ of the regions A_i		52	100
background contribution z_0 with standard		1293,2	45766
uncertainty $u(z_0)$ according to equation (C.12)		19,7	401
Results		**Example 4**	**Example 5**
measurand :	Y	A_M	I
quantity	symbol	value in Bq kg^{-1}	unit 1
primary measurement result	y	0,1346	26925
standard uncertainty associated with y	$u(y)$	0,0403	483
decision threshold	y^*	0,0619	747
measurement effect present ?	$y > y^*$?	yes	yes
detection limit	η^*	0,1279	1497
measurement procedure suitable ?	$\eta^* \leq \eta_\mathrm{r}$?	yes	–
lower confidence limit	η_l	0,0558	25978
upper confidence limit	η_u	0,2137	27871
best estimate the measurand	z	0,1347	26925
standard uncertainty associated with z	$u(z)$	0,0402	483
standardized chi-square	χ^2_s	–	1,21 (parabola), 4,18 (straight line)
chi-square condition fulfilled ?	$\chi^2_\mathrm{s} \leq k_{1-\delta/2}$?	–	yes (parabola), no (straight line)

Annex E
(informative)

Distribution function of the standardized normal distribution

The distribution function of the standardized normal distribution is defined by

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp(-v^2/2)\,\mathrm{d}v = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) \sum_{j=0}^{\infty} \frac{t^{2j+1}}{1 \cdot 3 \cdots (2j+1)} \tag{E.1}$$

and its quantile k_p for the probability p by $\Phi(k_p) = p$ [9]. The second expression in equation (E.1) can serve for the numerical calculation of $\Phi(t)$. The series in equation (E.1) converges for every t. Values of $\Phi(t)$ are presented in Table E.1. The relations $\Phi(-t) = 1 - \Phi(t)$ and $k_{1-p} = -k_p$ apply.

The quantile k_p of the standardized normal distribution can be calculated numerically as follows using the Newton iteration procedure: With an approximation t for k_p, an improved approximation t' results from

$$t' = t + \sqrt{2\pi} \exp(t^2/2)\,(p - \Phi(t)) \ . \tag{E.2}$$

The value $t = 0$ can be chosen as a starting approximation.

Table E.1: Distribution function $\Phi(t)$ of the standardized normal distribution

t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$	t	$\Phi(t)$
0,00	0,5000	0,60	0,7258	1,20	0,8849	1,80	0,9641	2,40	0,9918
0,02	0,5080	0,62	0,7324	1,22	0,8888	1,82	0,9656	2,42	0,9922
0,04	0,5160	0,64	0,7389	1,24	0,8925	1,84	0,9671	2,44	0,9927
0,06	0,5239	0,66	0,7454	1,26	0,8961	1,86	0,9686	2,46	0,9930
0,08	0,5319	0,68	0,7518	1,28	0,8997	1,88	0,9700	2,48	0,9934
0,10	0,5398	0,70	0,7580	1,30	0,9032	1,90	0,9713	2,50	0,9938
0,12	0,5478	0,72	0,7642	1,32	0,9066	1,92	0,9726	2,52	0,9941
0,14	0,5557	0,74	0,7704	1,34	0,9099	1,94	0,9738	2,54	0,9945
0,16	0,5636	0,76	0,7764	1,36	0,9131	1,96	0,9750	2,56	0,9948
0,18	0,5714	0,78	0,7823	1,38	0,9162	1,98	0,9762	2,58	0,9951
0,20	0,5793	0,80	0,7881	1,40	0,9192	2,00	0,9772	2,60	0,9953
0,22	0,5871	0,82	0,7939	1,42	0,9222	2,02	0,9783	2,62	0,9956
0,24	0,5948	0,84	0,7996	1,44	0,9251	2,04	0,9793	2,64	0,9959
0,26	0,6026	0,86	0,8051	1,46	0,9278	2,06	0,9803	2,66	0,9961
0,28	0,6103	0,88	0,8106	1,48	0,9306	2,08	0,9812	2,68	0,9963
0,30	0,6179	0,90	0,8159	1,50	0,9332	2,10	0,9821	2,70	0,9965
0,32	0,6255	0,92	0,8212	1,52	0,9357	2,12	0,9830	2,72	0,9967
0,34	0,6331	0,94	0,8264	1,54	0,9382	2,14	0,9838	2,74	0,9969
0,36	0,6406	0,96	0,8315	1,56	0,9406	2,16	0,9846	2,76	0,9971
0,38	0,6480	0,98	0,8365	1,58	0,9430	2,18	0,9854	2,78	0,9973
0,40	0,6554	1,00	0,8413	1,60	0,9452	2,20	0,9861	2,80	0,9974
0,42	0,6628	1,02	0,8461	1,62	0,9474	2,22	0,9868	2,90	0,9981
0,44	0,6700	1,04	0,8508	1,64	0,9495	2,24	0,9874	3,00	0,9986
0,46	0,6772	1,06	0,8554	1,66	0,9515	2,26	0,9881	3,10	0,9990
0,48	0,6844	1,08	0,8599	1,68	0,9535	2,28	0,9887	3,20	0,9993
0,50	0,6915	1,10	0,8643	1,70	0,9554	2,30	0,9893	3,30	0,9995
0,52	0,6985	1,12	0,8686	1,72	0,9573	2,32	0,9898	3,40	0,9997
0,54	0,7054	1,14	0,8729	1,74	0,9591	2,34	0,9904	3,50	0,9998
0,56	0,7123	1,16	0,8770	1,76	0,9610	2,36	0,9909	3,60	0,9998
0,58	0,7190	1,18	0,8810	1,78	0,9625	2,38	0,9913	3,80	0,9999
								≥4,00	1,0000

NOTE $k_p = t$ is the quantile for the probability $p = \Phi(t)$. The relations $\Phi(-t) = 1 - \Phi(t)$ and $k_{1-p} = -k_p$ apply.

Annex F

(informative)

Further terms

F.1 Background effect: measurement effect caused by the radiation background (for instance, from natural radiation sources)

F.2 Net effect: contribution of the possible radiation of a measurement object (for instance, of a radiation source or a radiation field) to the measurement effect

F.3 Gross effect: measurement effect caused by the background effect and the net effect

F.4 Shielding factor: factor describing the reduction of the background count rate by the shielding effect of the measurement object

F.5 Relaxation time constant: duration in which the output signal of a linear-scale ratemeter decreases to 1/e times the starting value after stopping the sequence of the input pulses

F.6 Background (in spectrometric measurements): number of the events of no interest in the region of a regarded line in the spectrum. The events can be due both to the background effect by the environmental radiation and also to the sample itself (for instance, from other lines).

Annex G

(informative)

Explanatory notes

G.1 General aspects of counting measurements

A measurement of ionizing radiation consists in general at least partially in counting electronic pulses induced by ionizing-radiation events. Such a measurement comprises several individual countings, but can also comprise sequences of individual countings. Examples are the countings on samples of radioactive material or on blanks, countings for the determination of the background effect or the countings in the individual channels of a multi-channel spectrum or in a temporal sequence in the same measurement situation. With each of the countings, either the measurement duration (time preselection) or the counting result (preselection of counts) can be fixed. On the basis of Bayesian statistics, all countings are treated in the same way as follows (see [7]).

The pulse number N of each of the countings is taken as a separate random variable. n is the counting result and t is the counting duration (measurement duration). N has the expectation value ϱt, where ϱ is the count rate or, with spectrum measurements, the spectral density. In the latter case, t is the channel width with respect to the assigned quantity, for instance, the particle energy. Either ϱ or ϱt is the measurand. It is assumed that dead-time and life-time effects, pile-up of the pulses, and instrumental instabilities can be neglected during counting and that all the counted pulses are induced by different ionizing-radiation events which are physically independent. The pulse number N then follows a Poisson distribution and the pulse numbers of all the countings are independent of each another.

Irrespective of whether n pulses are recorded in a measurement of a preselected duration (or of a fixed channel width) t (time preselection) or whether the measurement duration t needed for the counting of a preselected pulse number n is measured (preselection of counts), ϱt follows a gamma distribution, where ϱ is taken as a random variable. Then the best estimate r of the count rate (or spectral density) ϱ and the standard uncertainty $u(r)$ associated with r follow from

$$r = \mathsf{E}\,\varrho = n/t \ ; \quad u^2(r) = \mathsf{Var}\,(\varrho) = n/t^2 = r/t \ . \tag{G.1}$$

The case $n = 0$ results in $u(r) = 0$. This disappearing uncertainty of ϱ means that $\varrho = 0$ is exactly valid. But $u(r) = 0$ is an unrealistic result because, with a finite measurement duration, one can never be sure that exactly $\varrho = 0$ if no pulse happens to be recorded. This case can also lead to a zero denominator when the least-squares method according to DIN 1319-4 or [3] is applied and a division by $u^2(r)$ must be made. This shortcoming can be avoided by replacing all of the counting results n by $n + 1$ or, with a multi-channel spectrum, by a suitable combination of channels. Here, the measurement duration (or channel width) is assumed to be chosen from experience such that at least a few pulses can be expected if $\varrho > 0$.